Chapter 8: Acute Triangle Trigonometry

Getting Started, pages 422

1. a) ii) A triangle in which each interior angle is less than 90° is called **an acute triangle**.
b) iii) A triangle that contains a 90° angle is called **a right triangle**.
c) v) The longest side in a right triangle is called the **hypotenuse**.

d) iv) In a right triangle, the ratio $\frac{\text{adjacent}}{\text{hypotenuse}}$ is called **cosine**.
e) vi) The **Pythagorean theorem** describes the relationship between the three sides in a right triangle.

f) i) In a right triangle, the ratio $\frac{\text{opposite}}{\text{hypotenuse}}$ is called **sine**.
2. a)

Angle Measure	Reason
$a + 100° = 180°$ $a = 80°$	supplementary angles
$e + 30° + 90° = 180$ $e = 60°$	interior angles of a triangle sum to 180°
$b = e$ $b = 60°$	opposite angles
$b + d = 180°$ $60° + d = 180°$ $d = 120°$	supplementary angles
$a + b + c = 180°$ $80° + 60° + c = 180°$ $c = 40°$	interior angles of a triangle sum to 180°

b)

Angle Measure	Reason
$j = 75°$	parallel lines, so alternate angles are equal
$k + j + 25° = 180°$ $k + 75° + 25° = 180°$ $k = 80°$	interior angles of a triangle sum to 180°
$i = k = 80°$	parallel lines, so alternate angles are equal

c)

Angle Measure	Reason
$h + 35° + 90° = 180°$ $h = 55°$	interior angles of a triangle sum to 180°
$g = 35°$	similar triangles, so corresponding angles are equal
$f = h = 55°$	similar triangles, so corresponding angles are equal

d)

Angle Measure	Reason
$l + 70° + 55° = 180°$ $l = 55°$	supplementary angles
$l + m = 180°$ $55° + m = 180°$ $m = 125°$	parallel lines, so co-interior angles are supplementary
$n + m = 180°$ $n + 125° = 180°$ $n = 55°$	parallel lines, so co-interior angles are supplementary

3. a) AC is the longest side because it is opposite the right angle, the greatest angle. Determine $\angle C$ to see which side is the shortest. Since all interior angles of a triangle sum to 180°, then $\angle C = 180° - 90° - 48° = 42°$. AB is the shortest side because it is opposite the least angle, $\angle C = 42°$.
b) Since all interior angles of a triangle sum to 180°, then $\angle F = 180° - 65° - 50° = 65°$. Since both $\angle F$ and $\angle D$ are 65°, DE and EF are both the same length. Since $\angle F$ and $\angle D$ are the greatest angles, then both DE and EF are the longest sides. DF is, therefore, the shortest side.
4. a) $\angle B$. is a right angle or 90°. The interior angles in a triangle sum to 180°. So, the remaining angles, $\angle A$ and $\angle C$ must be less than 90°. The greatest angle is the right angle or $\angle B$. The least angle is opposite the shortest side, therefore, the least angle is $\angle A$.
b) The greatest angle is opposite the longest side. Therefore, the greatest angle is $\angle D$. The least angle is opposite the shortest side, therefore, the least angle is $\angle F$.
5. a) $\sin 55° = 0.8192$
b) $\cos 82° = 0.1392$
c) $\cos 24° = 0.9135$
d) $\sin 37° = 0.6018$

6. a)
$$\frac{5}{3} = \frac{x}{12}$$
$$12\left(\frac{5}{3}\right) = 12\left(\frac{x}{12}\right) \quad \text{Multiply both sides by 12.}$$
$$20 = x$$

b)
$$\frac{36}{x} = \frac{9}{2}$$
$$2x\left(\frac{36}{x}\right) = 2x\left(\frac{9}{2}\right) \quad \text{Multiply both sides by } 2x.$$
$$\frac{72}{9} = \frac{9x}{9} \quad \text{Divide both sides by 9.}$$
$$8 = x$$

c)
$$\sin 30° = \frac{x}{12}$$
$$12(\sin 30°) = 12\left(\frac{x}{12}\right) \quad \text{Multiply both sides by 12.}$$
$$12(0.5) = x$$
$$6 = x$$

d)
$$\cos 60° = \frac{25}{x}$$
$$x(\cos 60°) = x\left(\frac{25}{x}\right) \quad \text{Multiply both sides by } x.$$
$$x(\cos 60°) = 25$$
$$\frac{x(\cos 60°)}{\cos 60°} = \frac{25}{\cos 60°}$$
$$x = \frac{25}{\cos 60°}$$
$$x = 50$$

7. a) $\sin A = 0.5$

$\angle A = \sin^{-1}(0.5)$

$\angle A = 30°$

b) $\cos A = 0.5$

$\angle A = \cos^{-1}(0.5)$

$\angle A = 60°$

c) $\sin A = \frac{3}{4}$

$\angle A = \sin^{-1}\left(\frac{3}{4}\right)$

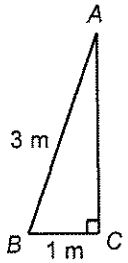
$\angle A \doteq 49°$

d) $\cos A = \frac{5}{8}$

$\angle A = \cos^{-1}\left(\frac{5}{8}\right)$

$\angle A \doteq 51°$

8. Draw a diagram to model the problem.



I can use the cosine ratio to calculate $\angle B$ *because* $\triangle ABC$ is a right triangle and I know the length of the sides opposite and adjacent to $\angle B$.

$\cos B = \frac{1}{3}$

$\angle B = \cos^{-1}\left(\frac{1}{3}\right)$

$\angle B \doteq 71°$

The angle that the board makes with the floor is about 71°.

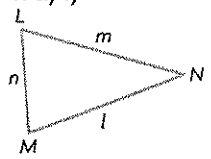
9. a) Since $AB \parallel CD$, $\angle A = \angle D$ and $\angle B = \angle C$ because they are alternate angles. Since the triangles have two pairs of corresponding equal angles, $\triangle ABG \sim \triangle DCG$.

b) From part a), we know the triangles are similar. Therefore, their corresponding sides are proportional. So the other ratios are $\frac{BG}{CG}$ and $\frac{AG}{DG}$.

10. Disagree. This is true for sine and cosine ratios because the hypotenuse (which is the denominator) is always longer than the adjacent or opposite side (which is the numerator). It isn't true for the tangent ratio: if the opposite side of an angle in a right triangle is longer than the adjacent side, the tangent ratio will be greater than one.

Lesson 8.1: Exploring the Sine Law, page 427

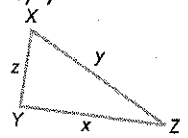
1. a) i)



ii) Because the triangle is an acute triangle, the following ratios are true:

$\frac{l}{\sin L} = \frac{m}{\sin M} = \frac{n}{\sin N}$

b) i)



ii) Because the triangle is an acute triangle, the following ratios are true:

$\frac{x}{\sin X} = \frac{y}{\sin Y} = \frac{z}{\sin Z}$

2. a) $\frac{w}{\sin 50°} = \frac{8.0}{\sin 60°}$

$w = \frac{8.0(\sin 50°)}{\sin 60°}$

$w \doteq 7.1$

b) $\frac{k}{\sin 43°} = \frac{9.5}{\sin 85°}$

$k = \frac{9.5(\sin 43°)}{\sin 85°}$

$k \doteq 6.5$

c) $\frac{6.0}{\sin M} = \frac{10.0}{\sin 72°}$

$\sin M(\sin 72°)\left(\frac{6.0}{\sin M}\right) = \sin M(\sin 72°)\left(\frac{10.0}{\sin 72°}\right)$

$6.0(\sin 72°) = 10.0(\sin M)$

$\frac{6.0(\sin 72°)}{10.0} = \sin M$

$\sin^{-1}(0.5706) \quad \angle M$

$34.8° = \angle M$

d) $\frac{12.5}{\sin Y} = \frac{14.0}{\sin 88°}$

$\sin Y(\sin 88°)\left(\frac{12.5}{\sin Y}\right) = \sin Y(\sin 88°)\left(\frac{14.0}{\sin 88°}\right)$

$12.5(\sin 88°) = 14.0(\sin Y)$

$\frac{12.5(\sin 88°)}{14.0} = \sin Y$

$0.8923 \quad \sin Y$

$\sin^{-1}(0.8923) = \angle Y$

$63.2° \doteq \angle Y$

3. Agree. Since $\frac{a}{\sin A} = \frac{b}{\sin B}$, multiplying both sides of the equation by $\sin A$ and $\sin B$ gives $a \sin B = b \sin A$.

4. To calculate the measure of an unknown side, you must know the measures of any two angles and the length of any other side. To calculate the measure of an unknown angle, you must know either the measures of the other two angles or the lengths of two sides and the measure of an angle that is opposite one of these sides. In both cases, you can use the sine law to determine the unknown value.

Lesson 8.2: Applying the Sine Law, page 432

1. According to the sine law,

$$\frac{q}{\sin Q} = \frac{r}{\sin R} = \frac{s}{\sin S}$$

2. a) I used the sine law to determine b because the triangle is acute and I knew an opposite side-angle pair, c and $\angle C$, and the angle opposite b, $\angle B$.

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\frac{b}{\sin 72°} = \frac{27.2}{\sin 43°}$$
$$b = \frac{27.2(\sin 72°)}{\sin 43°}$$
$$b = 37.9$$

Side b is 37.9 cm.

b) I used the sine law to determine $\angle C$ because the triangle is acute and I knew an opposite side-angle pair, b and $\angle B$, and the side length opposite $\angle C$, c.

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$
$$\frac{\sin C}{37.1} = \frac{\sin 44°}{29.5}$$
$$\sin C = \frac{37.1(\sin 44°)}{29.5}$$
$$\sin C \quad 0.8736$$
$$\angle C = \sin^{-1}(0.8736)$$
$$\angle C = 60.9°$$

3. a) If I calculate $\angle F$, then I can use the sine law to determine d, because I will know one opposite side-angle pair (f and $\angle F$) and the angle opposite d ($\angle D$).

$\angle F = 180° - 53° - 68°$ (sum of the angles in a triangle is 180°)
$= 59°$

$$\frac{d}{\sin D} = \frac{f}{\sin F}$$
$$\frac{d}{\sin 53°} = \frac{22.5}{\sin 59°}$$
$$d = \frac{22.5(\sin 53°)}{\sin 59°}$$
$$d \doteq 21.0$$

Side d is about 21.0 cm.

b) If I calculate $\angle C$, then I can use the sine law to determine a, because I will know one opposite side-angle pair (c and $\angle C$) and the angle opposite a ($\angle A$).

$\angle C = 180° - 40° - 60°$ (sum of the angles in a triangle is 180°)
$= 80°$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{a}{\sin 40°} = \frac{40.0}{\sin 80°}$$
$$a = \frac{40.0(\sin 40°)}{\sin 80°}$$
$$a \doteq 26.1$$

Side a is about 26.1 cm.

I can use the sine law to determine b, because I know one opposite side-angle pair (c and $\angle C$) and the angle opposite b ($\angle B$).

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\frac{b}{\sin 60°} = \frac{40.0}{\sin 80°}$$
$$b = \frac{40.0(\sin 60°)}{\sin 80°}$$
$$b \doteq 35.2$$

Side b is about 35.2 cm.

c) If I calculate $\angle Y$, I can use the sine law to determine y, because I will know one opposite side-angle pair (z and $\angle Z$) and the angle opposite y ($\angle Y$).

$\angle Y = 180° - 88° - 25°$ (sum of the angles in a triangle is 180°)
$= 67°$

$$\frac{y}{\sin Y} = \frac{z}{\sin Z}$$
$$\frac{y}{\sin 67°} = \frac{3.0}{\sin 25°}$$
$$y = \frac{3.0(\sin 67°)}{\sin 25°}$$
$$y \doteq 6.5$$

Side y is about 6.5 cm.

d) I can use the sine law to determine θ or $\angle N$ because I know one opposite side-angle pair (l and $\angle L$) and the side opposite θ or $\angle N$, n.

$$\frac{\sin N}{n} = \frac{\sin L}{l}$$
$$\frac{\sin N}{45.2} = \frac{\sin 29°}{24.4}$$
$$\sin N = \frac{45.2(\sin 29°)}{24.4}$$
$$\sin N \quad 0.8981$$
$$\angle N = \sin^{-1}(0.8981)$$
$$\angle N \doteq 64°$$

Since $\angle N = \theta$, then $\theta \doteq 64°$.

e) I can use the sine law to determine $\angle S$ or angle α, because I know one opposite side-angle pair (q and Q) and the side opposite α or $\angle S$, s.

$$\frac{\sin S}{s} = \frac{\sin Q}{q}$$
$$\frac{\sin S}{6.5} = \frac{\sin 50°}{5.0}$$
$$\sin S = \frac{6.5(\sin 50°)}{5.0}$$
$$\angle S = \sin^{-1}(0.9959)$$
$$\angle S \doteq 85° \text{ or } \alpha \doteq 85°$$
$$\angle R \doteq 180° - 50° - 85° \text{ (sum of angles in a triangle is 180°)}$$
$$\doteq 45°$$

or $\theta \doteq 45°$

f) I can use the sine law to determine θ or $\angle L$ because I know one opposite side-angle pair (k and $\angle K$) and the side opposite θ or $\angle L$, l.

$$\frac{\sin L}{l} = \frac{\sin K}{k}$$
$$\frac{\sin L}{2.9} = \frac{\sin 80°}{6.7}$$
$$\sin L = \frac{2.9(\sin 80°)}{6.7}$$
$$\sin L \doteq 0.4263$$
$$\angle L = \sin^{-1}(0.4263)$$
$$\angle L \doteq 25°$$

Since $\angle L = \theta$, then $\theta \doteq 25°$.

If I calculate α or $\angle J$, then I can use the sine law to determine j, because I will know one opposite side-angle pair (k and $\angle K$) and the angle opposite j, α, or $\angle J$.

$$\angle J \doteq 180° - 80° - 25° \text{ (sum of the angles is 180°)}$$
$$\doteq 75°$$

or $\alpha \doteq 75°$

$$\frac{j}{\sin J} = \frac{k}{\sin K}$$
$$\frac{j}{\sin 75°} = \frac{6.7}{\sin 80°}$$
$$j = \frac{6.7(\sin 75°)}{\sin 80°}$$
$$j \doteq 6.6$$

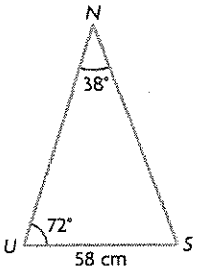
Side j is about 6.6 m.

4. If I calculate $\angle A$, then I can use the sine law to determine the length of the lake, c, because I will know one opposite side-angle pair (a and $\angle A$) and the angle opposite c ($\angle C$).

$$\angle A = 180° - 52° - 48° \text{ (sum of the angles is 180°)}$$
$$= 80°$$
$$\frac{c}{\sin C} = \frac{a}{\sin A}$$
$$\frac{c}{\sin 48°} = \frac{4340}{\sin 80°}$$
$$c = \frac{4340(\sin 48°)}{\sin 80°}$$
$$c \doteq 3275$$

Lake Lebarge is about 3275 m long.

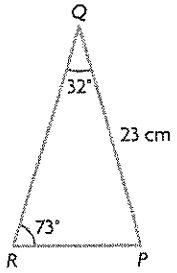
5. a) I can use the sine law to determine u, because I know one opposite side-angle pair (n and $\angle N$) and the angle opposite u ($\angle U$).



$$\frac{u}{\sin U} = \frac{n}{\sin N}$$
$$\frac{u}{\sin 72°} = \frac{58}{\sin 38°}$$
$$u = \frac{58(\sin 72°)}{\sin 38°}$$
$$u \doteq 90$$

Side u is about 90 cm.

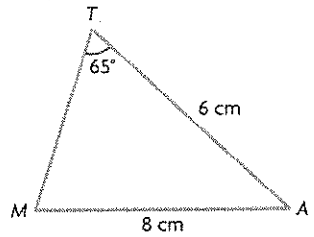
b) I can use the sine law to determine q, because I know one opposite side-angle pair (r and $\angle R$) and the angle opposite u ($\angle Q$).



$$\frac{q}{\sin Q} = \frac{r}{\sin R}$$
$$\frac{q}{\sin 32°} = \frac{23}{\sin 73°}$$
$$q = \frac{23(\sin 32°)}{\sin 73°}$$
$$q \doteq 13$$

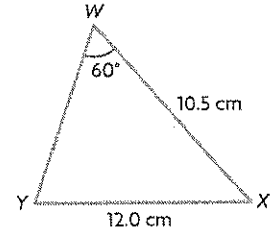
Side q is about 13 cm.

c) I can use the sine law to determine $\angle M$, because I know one opposite side-angle pair (t and $\angle T$) and the side length opposite $\angle M$, m.



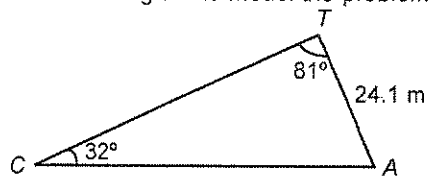
$$\frac{\sin M}{m} = \frac{\sin T}{t}$$
$$\frac{\sin M}{6} = \frac{\sin 65°}{8}$$
$$\sin M = \frac{6(\sin 65°)}{8}$$
$$\sin M \quad 0.6797$$
$$\angle M = \sin^{-1}(0.6797)$$
$$\angle M \doteq 43°$$

d) I can use the sine law to determine $\angle Y$, because I know one opposite side-angle pair (w and $\angle W$) and the side length opposite $\angle Y$, y.



$$\frac{\sin Y}{y} = \frac{\sin W}{w}$$
$$\frac{\sin Y}{10.5} = \frac{\sin 60°}{12.0}$$
$$\sin Y = \frac{10.5(\sin 60°)}{12.0}$$
$$\sin Y \quad 0.7578$$
$$\angle Y = \sin^{-1}(0.7578)$$
$$\angle Y \doteq 49°$$

6. Draw a diagram to model the problem.



Determine $\angle A$.

$\angle A = 180° - 81° - 32°$ (sum of the angles of a triangle is 180°)
$= 67°$

I can use the sine law to determine a, because I know one opposite side-angle pair (c and $\angle C$) and the angle opposite a ($\angle A$).

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{a}{\sin 67°} = \frac{24.1}{\sin 32°}$$
$$a = \frac{24.1(\sin 67°)}{\sin 32°}$$
$$a \doteq 41.9$$

Side a is about 41.9 cm.

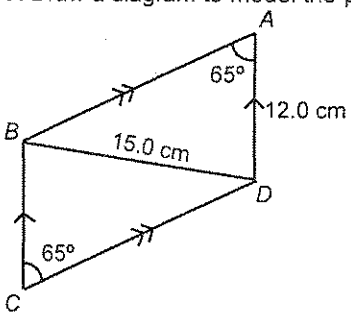
I can use the sine law to determine u, because I know one opposite side-angle pair (n and $\angle N$) and the angle opposite u ($\angle U$).

I can use the sine law to determine t, because I know one opposite side-angle pair (c and $\angle C$) and the angle opposite t ($\angle T$).

$$\frac{t}{\sin T} = \frac{c}{\sin C}$$
$$\frac{t}{\sin 81°} = \frac{24.1}{\sin 32°}$$
$$t = \frac{24.1(\sin 81°)}{\sin 32°}$$
$$t \doteq 44.9$$

Side t is about 44.9 m.

7. Draw a diagram to model the problem.



I can use the sine law to determine $\angle ABD$ because I know one opposite side-angle pair (a and $\angle A$) and the side opposite $\angle ABD$ (b).

$$\frac{\sin ABD}{b} = \frac{\sin A}{a}$$
$$\frac{\sin ABD}{12.0} = \frac{\sin 65°}{15.0}$$
$$\sin ABD = \frac{12.0(\sin 65°)}{15.0}$$
$$\angle ABD = \sin^{-1}\left(\frac{12.0(\sin 65°)}{15.0}\right)$$
$$\angle ABD \doteq 46.5°$$

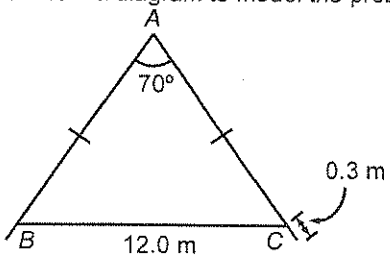
If I calculate $\angle ADB$, I can use the sine law to determine d because I will know one opposite side-angle pair (a and $\angle A$) and the angle opposite d ($\angle ADB$).

$\angle ADB \doteq 180° - 65° - 46.5°$ (sum of angles is 180°)
$\doteq 68.5°$

$$\frac{d}{\sin ADB} = \frac{a}{\sin A}$$
$$\frac{d}{\sin 68.5°} = \frac{15.0}{\sin 65°}$$
$$d = \frac{15.0(\sin 68.5°)}{\sin 65°}$$
$$d = 15.4$$

The long sides of the parallelogram are 15.4 cm.

8. Draw a diagram to model the problem.



If I determine $\angle C$, I can use the sine law to determine b because I will know one opposite side-angle pair (a and $\angle A$) and the angle opposite b ($\angle B$).

$\angle C = \angle A = 55°$ ($\triangle ABC$ is isosceles and $c = a$)

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$
$$\frac{b}{\sin 55°} = \frac{12.0}{\sin 70°}$$
$$b = \frac{12.0(\sin 55°)}{\sin 70°}$$
$$b \doteq 10.5$$

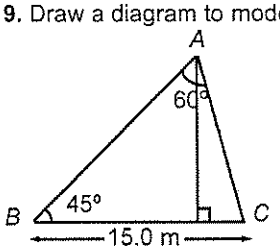
AC is about 10.5 m long.

Add b to the rafter extension.

Length of rafter $\doteq b + 0.3$
$\doteq 10.5 + 0.3$
$\doteq 10.8$

The rafters are about 10.8 m long.

9. Draw a diagram to model the problem.



I can use the sine law to determine wire b because I know one opposite side-angle pair (a and $\angle A$) and the angle opposite b ($\angle B$).

$$\frac{AC}{\sin B} = \frac{BC}{\sin A}$$

$$\frac{AC}{\sin 45°} = \frac{15.0}{\sin 60°}$$

$$AC = \frac{15.0(\sin 45°)}{\sin 60°}$$

$$AC \doteq 12.2$$

If I determine $\angle C$, I can use the sine law to determine wire c because I will know one opposite side-angle pair (a and $\angle A$) and the angle opposite c ($\angle C$).

$\angle C = 180° - 60° - 45°$ (sum of angles of triangle)

$= 75°$

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

$$\frac{c}{\sin 75°} = \frac{15.0}{\sin 60°}$$

$$c = \frac{15.0(\sin 75°)}{\sin 60°}$$

$$c \doteq 16.7$$

Let D represent the base of the pole. Now that I know c, I can use the sine ratio for 45° to determine the height of the pole, AD, because $\triangle ABD$ is a right triangle.

$$\sin B = \frac{AD}{c}$$

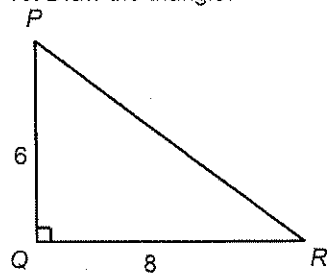
$$\sin 45° = \frac{AD}{16.7}$$

$$16.7(\sin 45°) = AD$$

$$11.8 \doteq AD$$

The wires are about 12.2 m and about 16.7 m, and the pole is about 11.8 m.

10. Draw the triangle.



Method 1: Use the tangent ratio to determine $\angle P$.

$$\tan P = \frac{QR}{PQ}$$

$$\tan P = \frac{8}{6}$$

$$\angle P = \tan^{-1}\left(\frac{8}{6}\right)$$

$$\angle P \doteq 53°$$

Method 2: Use the Pythagorean theorem to determine q and then use the sine ratio to determine $\angle P$.

$$q^2 = r^2 + p^2$$

$$q^2 = 6^2 + 8^2$$

$$q^2 = 100$$

$$q = 10$$

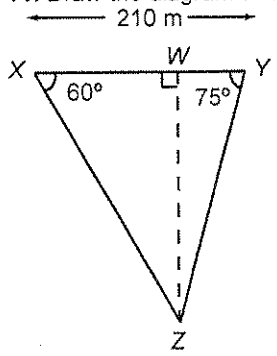
$$\sin P = \frac{p}{q}$$

$$\sin P = \frac{8}{10}$$

$$\angle P = \sin^{-1}\left(\frac{8}{10}\right)$$

$$\angle P \doteq 53°$$

11. Draw the diagram and label the vertices.



If I calculate $\angle XZY$, I can use the sine law to determine XZ because I will know one opposite side-angle pair (XY and $\angle XZY$) and the angle opposite XZ ($\angle XYZ$).

$\angle XZY = 180° - 60° - 75°$ (sum of angles of a triangle)

$\angle XZY = 45°$

$$\frac{XZ}{\sin XYZ} = \frac{XY}{\sin XZY}$$

$$\frac{XZ}{\sin 75°} = \frac{210}{\sin 45°}$$

$$XZ = \frac{210(\sin 75°)}{\sin 45°}$$

$$XZ \doteq 286.9$$

Now that I know XZ, I can use the sine ratio of 60° to determine WZ because $\triangle WXZ$ is a right triangle.

$$\sin X = \frac{WZ}{XZ}$$

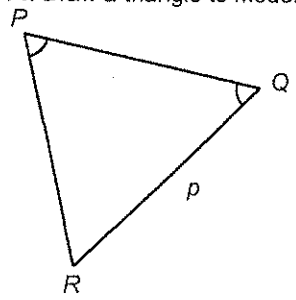
$$\sin 60° \doteq \frac{WZ}{286.9}$$

$$286.9(\sin 60°) = WZ$$

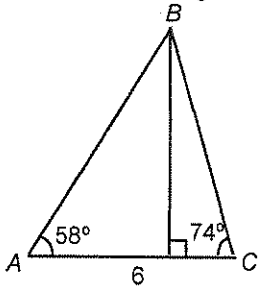
$$248 = WZ$$

The gorge is 248 m deep.

12. Answers may vary. For example,
a) The sine law can be used to solve a triangle if the measures of any two angles and the length of any side are known, or if the lengths of two sides and the measure of an angle opposite one of these sides are known.
b) The sine law cannot be used to solve a triangle if the lengths of all three sides are known but no angle measures are known, or if the measures of all three angles are known but no side lengths are known, or if the lengths of two sides and the measure of an angle that is not opposite one of these sides are known.
13. Agree. The given angle measure is not opposite any given side, so a complete ratio of the sine law cannot be formed. You must be able to write a proportion with two ratios that includes only one unknown value.
14. Draw a triangle to model the problem.



Calculate $\angle R$ using the fact that the sum of the interior angles in a triangle is 180°. Calculate side lengths q and r using the sine law.
15. Draw the triangle. Include the height of the triangle.



If I calculate $\angle B$, I can use the sine law to determine c because I will know one opposite side-angle pair (b and $\angle B$) and the angle opposite c ($\angle C$).
$\angle B = 180° - 58° - 74°$ (sum of the angles in a triangle)
$\angle B = 48°$

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$
$$c = \frac{b}{\sin B} \times \sin C$$
$$c = \frac{6}{\sin 48°} \times \sin 74°$$
$$c \doteq 7.76$$

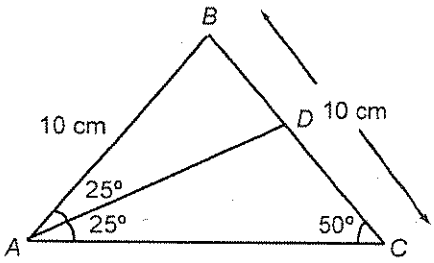
Let h represent the height of the triangle. Now that I know c, I can determine h, the height of the triangle, using the sine ratio.

$$\frac{h}{c} = \sin A$$
$$h = c \sin A$$
$$h \doteq 7.76(\sin 58°)$$
$$h \doteq 6.58$$

Now that I know the base, b, and height, h, of the triangle, I can calculate its area.

$$A = \frac{1}{2}bh$$
$$\doteq \frac{1}{2}(6)(6.58)$$
$$= 19.7$$

The area is 19.7 square units.
16. Draw the triangle and label the vertices.



To use the sine law to determine AD in $\triangle ABD$, I need to determine $\angle ABD$ and then use it to determine $\angle ADB$.
Since $\angle ABD = \angle B$ in $\triangle ABC$, I can subtract $\angle A$ and $\angle C$ from 180°.
$\angle ABD = 180° - 50° - 50°$ (sum of angles in a triangle)
$\angle ABD = 80°$
Use $\angle ABD$ to determine $\angle ADB$.
$\angle ADB = 180° - 80° - 25°$ (sum of angles in a triangle)
$\angle ADB = 75°$
Use the sine law to determine AD.

$$\frac{AD}{\sin ABD} = \frac{AB}{\sin ADB}$$
$$AD = \frac{AB}{\sin ADB} \times \sin ABD$$
$$AD = \frac{10}{\sin 75°} \times \sin 80°$$
$$AD \doteq 10.2$$

The line segment is about 10.2 cm.

17. a) $\frac{b}{\sin B}$ or $\frac{c}{\sin C}$

b) $\frac{\sin A}{a} = \frac{\sin B}{b}$

$(a)(b)\frac{\sin A}{a} = (a)(b)\frac{\sin B}{b}$

$b\sin A = a\sin B$

$\frac{b\sin A}{b\sin B} = \frac{a\sin B}{b\sin B}$

$\frac{\sin A}{\sin B} = \frac{a}{b}$

c) $\frac{\sin A}{a} = \frac{\sin C}{c}$

$(a)(c)\frac{\sin A}{a} = (a)(c)\frac{\sin C}{c}$

$c\sin A = a\sin C$

$\frac{c\sin A}{c\sin C} = \frac{a\sin C}{c\sin C}$

$\frac{\sin A}{\sin C} = \frac{a}{c}$

d) $\frac{\sin A}{a} = \frac{\sin C}{c}$

$(a)(c)\frac{\sin A}{a} = (a)(c)\frac{\sin C}{c}$

$c\sin A = a\sin C$

$\frac{c\sin A}{c\sin A} = \frac{a\sin C}{c\sin A}$

$1 = \frac{a\sin C}{c\sin A}$

Mid-Chapter Review, page 436

1. $\frac{x}{\sin X} = \frac{y}{\sin Y} = \frac{z}{\sin Z}$

2. Answers may vary, e.g.,
For right triangles, you can use the primary trigonometric ratios or the Pythagorean theorem. For acute triangles, you can only use the sine law.

3. Disagree. $\frac{d}{\sin D} = \frac{f}{\sin F}$ or $d \sin F = f \sin D$, but neither of these is equivalent to $\frac{d}{\sin F} = \frac{f}{\sin D}$.

4. a) I can use the sine law to determine θ because I know one opposite side-angle pair (50° and 4.5 cm) and the side length opposite θ (4.0 cm).

$\frac{\sin\theta}{4.0} = \frac{\sin 50°}{4.5}$

$\sin\theta = \frac{4.0(\sin 50°)}{4.5}$

$\sin\theta \quad 0.6809$

$\theta = \sin^{-1}(0.6809)$

$\theta \doteq 43°$

If I calculate the angle opposite x, I can use the sine law to determine x because I will know one opposite side-angle pair (50° and 4.5 cm) and the angle opposite x (87°).

Angle opposite $x = 180° - 50° - 43°$ (sum of angles in a triangle)
$= 87°$

$\frac{x}{\sin 87°} = \frac{4.5}{\sin 50°}$

$x = \frac{4.5(\sin 87°)}{\sin 50°}$

$x \doteq 5.9$

Side x is about 5.9 cm and θ is about 60°.

b) Determine θ by subtracting the two other known angles in the triangle from 180.

$\theta = 180° - 55° - 63°$ (sum of angles of a triangle is 180°)

$\theta = 62°$

Now that I know $\theta = 62°$, I can use the sine law to determine x because I know one opposite side-angle pair (62° and 10.5 cm) and the angle opposite x (63°).

$\frac{x}{\sin 63°} = \frac{10.5}{\sin 62°}$

$x = \frac{10.5(\sin 63°)}{\sin 62°}$

$x \doteq 10.6$

Now that I know $\theta = 62°$, I can use the sine law to determine y because I know one opposite side-angle pair (62° and 10.5 cm) and the angle opposite y (55°).

$\frac{y}{\sin 55°} = \frac{10.5}{\sin 62°}$

$y = \frac{10.5(\sin 55°)}{\sin 62°}$

$y \doteq 9.7$

Side x is about 10.6 cm and side y is about 9.7 cm.

5. I can use the sine law to determine b because I know one opposite side-angle pair (70° and 15 cm) and the angle opposite b (50°).

$\frac{b}{\sin B} = \frac{a}{\sin A}$

$\frac{b}{\sin 50°} = \frac{15}{\sin 70°}$

$b = \frac{15(\sin 50°)}{\sin 70°}$

$b \doteq 12.2$

Determine $\angle C$ by subtracting the two other known angles in the triangle from 180.

$\angle C = 180° - 70° - 50°$ (sum of angles of a triangle)

$\angle C = 60°$

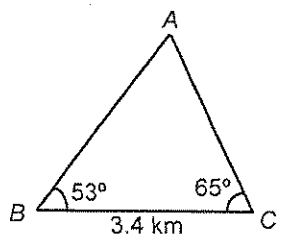
Now that I know $\angle C = 60°$, I can use the sine law to determine c because I know one opposite side-angle pair (70° and 15 cm) and the angle opposite c (60°).

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$
$$\frac{c}{\sin 60°} = \frac{15}{\sin 70°}$$
$$c = \frac{15(\sin 60°)}{\sin 70°}$$
$$c \doteq 13.8$$

Side b is about 12.2 cm, side c is about 13.8 cm, and $\angle C$ is 60°.

6. $\angle X$ or $\angle Z$ must be known since they are opposite x and z.

7. a) Draw a diagram to model the problem.



I know that the greatest angle is opposite the greatest side. Since I just need to compare the sides from the right tower to the fire, b, and from the left tower to the fire, c, I will only compare the angles between each tower and the other ($\angle B$ and $\angle C$). Because side b is opposite the lesser angle, side b is less than side c. Therefore, the tower on the right is closer to the fire.

b) Refer to the diagram from part a).

If I calculate $\angle A$, I can use the sine law to determine c because I will know one opposite side-angle pair ($\angle A$ and a) and the angle opposite c (65°).

$\angle A = 180° - 70° - 50°$ (sum of angles in a triangle is 180°)

$\angle A = 60°$

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$
$$\frac{c}{\sin 65°} = \frac{3.4}{\sin 60°}$$
$$c = \frac{3.4(\sin 65°)}{\sin 60°}$$
$$c \doteq 3.6$$

I can use the sine law to determine b because I know one opposite side-angle pair ($\angle A$ and a) and the angle opposite b (53°).

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$
$$\frac{b}{\sin 53°} = \frac{3.4}{\sin 60°}$$
$$b = \frac{3.4(\sin 53°)}{\sin 60°}$$
$$b \doteq 3.1$$

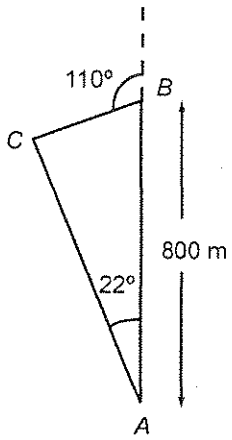
The tower on the right, which is closer to the fire, is about 3.1 km away from it.

8. Draw a diagram to model the problem.

Let A represent the paddlers' initial position.

Let B represent the paddlers' second position.

Let C represent the campsite.



If I calculate $\angle C$, I can use the sine law to determine a because I will know one opposite side-angle pair ($\angle C$ and c) and the angle opposite a.

$\angle B + 110° = 180°$ (supplementary angles)

$\angle B = 70°$

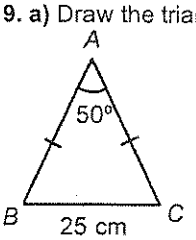
$\angle C = 180° - 70° - 22°$ (sum of angles is 180°)

$\angle C = 88°$

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$
$$\frac{800}{\sin 88°} = \frac{a}{\sin 22°}$$
$$\sin 22°\left(\frac{800}{\sin 88°}\right) = a$$
$$300 \doteq a$$

At the second sighting, the campsite is about 300 m away.

9. a) Draw the triangle and label the vertices.



If I calculate $\angle B$, I can use the sine law to determine b because I will know one opposite side-angle pair ($\angle A$ and a) and the angle opposite b.

Since $\angle B + \angle C = 180° - 50° = 130°$ (sum of angles in a triangle = 180°) and $\angle B = \angle C$ (because $b = c$), then $\angle B = 130° \div 2 = 65°$.

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$
$$\frac{b}{\sin 65°} = \frac{25}{\sin 50°}$$
$$b = \frac{25(\sin 65°)}{\sin 50°}$$
$$b \doteq 29.6$$

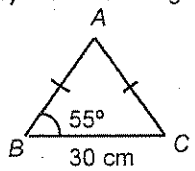
Since the triangle is isosceles, $c = b = 29.6$ cm.

Calculate the perimeter.

Perimeter $= a + b + c$
$\doteq 25 + 29.6 + 29.6$
$\doteq 84$

The perimeter is about 84 cm.

b) Draw he triangle and label the vertices.



If I calculate $\angle A$, I can use the sine law to determine b because I will know one opposite side-angle pair ($\angle A$ and a) and the angle opposite b.

Since $\angle C = \angle B = 55°$ (because $b = c$) then $\angle A = 180° - 55° + 55° = 70°$ (sum of angles in a triangle = 180°).

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

$$\frac{b}{\sin 55°} = \frac{30}{\sin 70°}$$

$$b = \frac{30(\sin 55°)}{\sin 70°}$$

$$b \doteq 26.2$$

Since the triangle is isosceles, $c = b = 26.2$ cm.

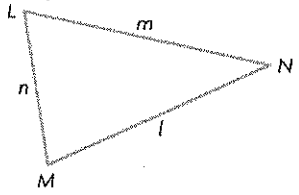
Calculate the perimeter.

Perimeter $= a + b + c$
$\doteq 30 + 26.2 + 26.2$
$\doteq 82$

The perimeter is about 82 cm.

Lesson 8.3: Exploring the Cosine Law, page 438

1. a)

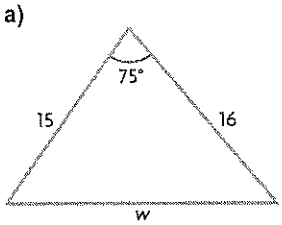


b)

$l^2 = m^2 + n^2 - 2mn \cos L$
$m^2 = l^2 + n^2 - 2ln \cos M$
$n^2 = l^2 + m^2 - 2lm \cos N$

2. Answers may vary, e.g.

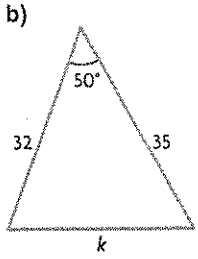
a)



$w^2 = 15^2 + 16^2 - 2(15)(16)\cos 75°$
$w^2 \doteq 356.7669$
$w \doteq \sqrt{356.7669}$
$w \doteq 18.9$

Side w is about 19 units.

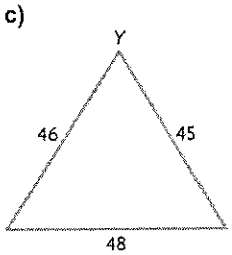
b)



$k^2 = 32^2 + 35^2 - 2(32)(35)\cos 50°$
$k^2 \doteq 809.1558$
$k \doteq \sqrt{809.1558}$
$k \doteq 28.4$

Side k is about 28 units.

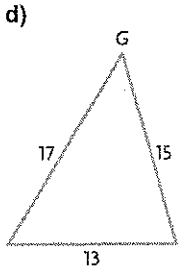
c)



$48^2 = 46^2 + 45^2 - 2(46)(45)\cos Y$
$2304 = 2116 + 2025 - 4140 \cos Y$
$4140 \cos Y = 4141 - 2304$
$\cos Y = \frac{1837}{4140}$
$\angle Y = \cos^{-1}(0.44)$
$\angle Y \doteq 63.9°$

$\angle Y$ is about 64°.

d)



$13^2 = 17^2 + 15^2 - 2(17)(15)\cos G$
$169 = 289 + 224 - 510 \cos G$
$510 \cos G = 514 - 169$
$\cos G = \frac{345}{510}$
$\angle G = \cos^{-1}\left(\frac{345}{510}\right)$
$\angle G \doteq 47.4°$

$\angle G$ is about 47°.

3. Answers may vary, e.g.,

a) If an angle measure needs to be determined, the cosine ratio can be determined quickly and the angle measure can be determined using $\cos^{-1}$.

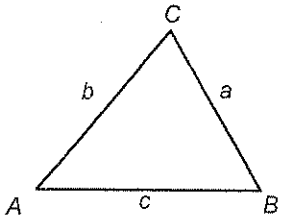
b)
$$p^2 = q^2 + r^2 - 2qr\cos P$$
$$p^2 - q^2 - r^2 = -2qr\cos P$$
$$\frac{p^2 - q^2 - r^2}{-2qr} = \frac{-2qr\cos P}{-2qr}$$
$$\frac{q^2 + r^2 - p^2}{2qr} = \cos P$$

c)
$$q^2 = p^2 + r^2 - 2pr\cos Q$$
$$q^2 - p^2 - r^2 = -2pr\cos Q$$
$$\frac{q^2 - p^2 - r^2}{-2pr} = \frac{-2pr\cos Q}{-2pr}$$
$$\frac{p^2 + r^2 - q^2}{2pr} = \cos Q$$

4. a) I would need to know the angle opposite the unknown side length and the other two sides.
b) I would need to know all three side lengths.
5. The square of a side length equals the sum of the squares of the other two side lengths minus twice the product of the other two side lengths and the cosine of the angle opposite the first side length.

Curious Math, page 439

1. Answers may vary, e.g.,



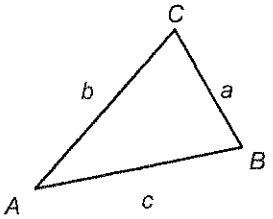
2. Answers may vary, e.g.,

$\angle A = 50°$ $a = 2.50$ cm
$\angle B = 60°$ $b = 2.80$ cm
$\angle C = 70°$ $c = 3.00$ cm

3. Answers may vary, e.g.,
$$\frac{a-b}{a+b} = \frac{2.50 - 2.80}{2.50 + 2.80}$$
$$= \frac{-0.30}{5.30}$$
$$\doteq -0.06$$

4. Answers may vary, e.g.,
$$\frac{\tan\left(\frac{1}{2}(A-B)\right)}{\tan\left(\frac{1}{2}(A+B)\right)} = \frac{\tan\left(\frac{1}{2}(50° - 60°)\right)}{\tan\left(\frac{1}{2}(50° + 60°)\right)}$$
$$= \frac{\tan\left(\frac{1}{2}(-10°)\right)}{\tan\left(\frac{1}{2}(110°)\right)}$$
$$= \frac{\tan -5°}{\tan 55°}$$
$$= -0.06$$
This is the same value from question 3.

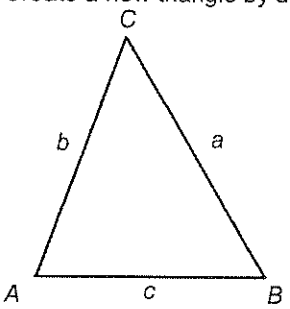
5. Answers may vary, e.g.,
It seems that the tangent law could be
$$\frac{a-b}{a+b} = \frac{\tan\left(\frac{1}{2}(A-B)\right)}{\tan\left(\frac{1}{2}(A+B)\right)}.$$
We can create a new triangle by dragging vertex B.



$\angle A = 38°$ $a = 1.82$ cm
$\angle B = 72°$ $b = 2.81$ cm
$\angle C = 70°$ $c = 2.75$ cm

$$\frac{a-b}{a+b} = \frac{1.82 - 2.81}{1.82 + 2.81}$$
$$= \frac{-0.99}{4.63}$$
$$\doteq -0.21$$

$$\frac{\tan\left(\frac{1}{2}(A-B)\right)}{\tan\left(\frac{1}{2}(A+B)\right)} = \frac{\tan\left(\frac{1}{2}(38° - 72°)\right)}{\tan\left(\frac{1}{2}(38° + 72°)\right)}$$
$$= \frac{\tan\left(\frac{1}{2}(-34°)\right)}{\tan\left(\frac{1}{2}(110°)\right)}$$
$$= \frac{\tan -17°}{\tan 55°}$$
$$\doteq -0.21$$
The result still holds for this triangle.
Create a new triangle by dragging vertex C.



$\angle A = 70°$ $a = 3.65$ cm
$\angle B = 60°$ $b = 3.35$ cm
$\angle C = 50°$ $c = 3.00$ cm

$$\frac{a-b}{a+b} = \frac{3.65 - 3.35}{3.65 + 3.35}$$
$$= \frac{0.30}{7.00}$$
$$\doteq 0.04$$

$$\frac{\tan\left(\frac{1}{2}(A-B)\right)}{\tan\left(\frac{1}{2}(A+B)\right)} = \frac{\tan\left(\frac{1}{2}(70°-60°)\right)}{\tan\left(\frac{1}{2}(70°+60°)\right)}$$
$$= \frac{\tan\left(\frac{1}{2}(10°)\right)}{\tan\left(\frac{1}{2}(130°)\right)}$$
$$= \frac{\tan 5°}{\tan 65°}$$
$$\doteq 0.04$$

This result holds true for all three triangles we've tried.

6. a) $\frac{b-c}{b+c} = \frac{\tan\left(\frac{1}{2}(B-C)\right)}{\tan\left(\frac{1}{2}(B+C)\right)}$

b) $\frac{a-c}{a+c} = \frac{\tan\left(\frac{1}{2}(A-C)\right)}{\tan\left(\frac{1}{2}(A+C)\right)}$

Lesson 8.4: Applying the Cosine Law, page 443

1. a) No. Another side length, specifically b, is required because to use the cosine law, you need to know two sides and the angle between them.

b) Yes. The lengths of two sides and the measure of the angle between them are given, so the cosine law can be used.

2. a) I can use the cosine law to solve for x because $\triangle ABC$ is acute and I know two side lengths, y and z, and the angle between them, $\angle X$.

$$x^2 = 18^2 + 15^2 - 2(18)(15)\cos 46°$$
$$x^2 = 173.8845$$
$$x = \sqrt{173.8845}$$
$$x \doteq 13.2$$

Side x is about 13.2 cm.

b) I can use the cosine law to solve for $\angle P$ because $\triangle PQR$ is acute and I know all three side lengths.

$$5.9^2 = 6.2^2 + 2.3^2 - 2(6.2)(2.3)\cos P$$
$$34.81 = 38.44 + 5.29 - 28.52\cos P$$
$$34.81 - 43.73 = -28.52\cos P$$
$$-8.92 = -28.52\cos P$$
$$\frac{-8.92}{-28.52} = \frac{-28.52\cos P}{-28.52}$$
$$0.3128 \doteq \cos P$$
$$\cos^{-1}(0.3128) = \angle P$$
$$72° \doteq \angle P$$

$\angle P$ is about 72°.

3. a) I can use the cosine law to solve for b because $\triangle ABC$ is acute and I know two side lengths, a and c, and the angle between them, $\angle B$.

$$b^2 = 10.5^2 + 9.5^2 - 2(10.5)(9.5)\cos 40°$$
$$b^2 \doteq 47.674\,133\,6$$
$$b \doteq \sqrt{47.674\,133\,6}$$
$$b \doteq 6.9$$

Side b is about 6.9 cm.

b) I can use the cosine law to solve for e because $\triangle DEF$ is acute and I know two side lengths, d and f, and the angle between them, $\angle E$.

$$e^2 = 13^2 + 11^2 - 2(13)(11)\cos 75°$$
$$e^2 \doteq 215.977\,753\,1$$
$$e \doteq \sqrt{215.977\,753\,1}$$
$$e \doteq 14.7$$

Side e is about 14.7 cm.

4. a) I can use the cosine law to solve for $\angle P$ because $\triangle PQR$ is acute and I know all three side lengths.

$$2.2^2 = 3.9^2 + 3.5^2 - 2(3.9)(3.5)\cos P$$
$$4.84 = 15.21 + 12.25 - 27.3\cos P$$
$$4.84 - 15.21 - 12.25 = -27.3\cos P$$
$$-22.62 = -27.3\cos P$$
$$\frac{-22.62}{-27.3} = \cos P$$
$$0.8286 \doteq \cos P$$
$$\cos^{-1}(0.8286) = \angle P$$
$$34° = \angle P$$

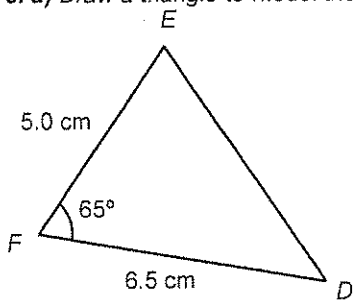
To the nearest degree, $\angle P$ is 34°.

b) I can use the cosine law to solve for $\angle Z$ because $\triangle XYZ$ is acute and I know all three side lengths.

$$2.9^2 = 2.6^2 + 2.2^2 - 2(2.6)(2.2)\cos Z$$
$$8.41 = 6.76 + 4.84 - 11.44\cos Z$$
$$8.41 - 6.76 - 4.84 = -11.44\cos Z$$
$$-3.19 = -11.44\cos Z$$
$$\frac{-3.19}{-11.44} = \cos Z$$
$$0.2788 \doteq \cos Z$$
$$\cos^{-1}(0.2788) = \angle Z$$
$$74° = \angle Z$$

To the nearest degree, $\angle Z$ is 74°.

5. a) Draw a triangle to model the problem.



I can use the cosine law to solve for f because $\triangle DEF$ is acute and I know two side lengths, d and e, and the angle between them, $\angle F$.

$$f^2 = 5.0^2 + 6.5^2 - 2(5.0)(6.5)\cos 65°$$
$$f^2 \doteq 39.7798$$
$$f \doteq \sqrt{39.7798}$$
$$f \doteq 6.3$$

Now that I know f, I can use the cosine law to solve for $\angle D$ because I know all three side lengths

$$5.0^2 = 6.3^2 + 6.5^2 - 2(6.3)(6.5)\cos D$$
$$25.0 = 39.69 + 42.25 - 81.9\cos D$$
$$25.0 - 39.69 - 42.25 = -81.9\cos D$$
$$-56.94 = -81.9\cos D$$
$$\frac{-56.94}{-81.9} = \cos D$$
$$0.6952 \doteq \cos D$$
$$\cos^{-1}(0.6952) = \angle D$$
$$45.9^\circ \doteq \angle D$$

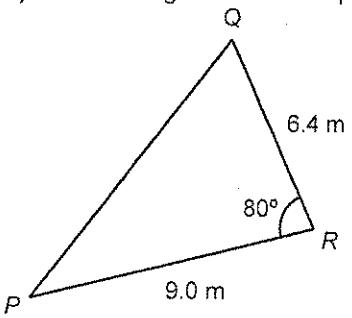
Now that I know $\angle D$, I can subtract the two known angles from 180° to determine $\angle E$.

$\angle E = 180^\circ - 45.9^\circ - 65^\circ$ (sum of angles is 180°)

$\angle E = 69.1^\circ$

Side f is about 6.3 cm, $\angle D$ is about 46°, and $\angle E$ is about 69°.

b) Draw a triangle to model the problem.



I can use the cosine law to solve for r because $\triangle PQR$ is acute and I know two side lengths, p and q, and the angle between them, $\angle R$.

$$r^2 = 6.4^2 + 9.0^2 - 2(6.4)(9.0)\cos 80^\circ$$
$$r^2 \doteq 101.9557$$
$$r \doteq \sqrt{101.9557}$$
$$r \doteq 10.1$$

Now that I know r, I can use the cosine law to solve for $\angle Q$ because $\triangle PQR$ is acute and I know all three side lengths.

$$9.0^2 = 10.1^2 + 6.4^2 - 2(10.1)(6.4)\cos Q$$
$$81.0 = 102.01 + 40.96 - 129.28\cos Q$$
$$81.0 - 102.01 - 40.96 = -129.28\cos Q$$
$$-61.97 = -129.28\cos Q$$
$$\frac{-61.97}{-129.28} = \cos Q$$
$$0.4793 \doteq \cos Q$$
$$\cos^{-1}(0.4793) = \angle Q$$
$$61.4^\circ \doteq \angle Q$$

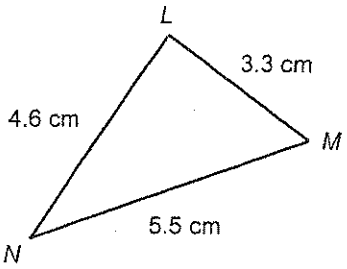
Now that I know $\angle Q$, I can subtract the two known angles from 180° to determine $\angle P$.

$\angle P = 180^\circ - 61.4^\circ - 80^\circ$ (sum of angles in a triangle is 180°)

$\angle P = 38.6^\circ$

Side r is about 10.1 m, $\angle Q$ is about 61°, and $\angle P$ is about 39°.

c) Draw a triangle to model the problem.



I can use the cosine law to solve for $\angle L$ because $\triangle LMN$ is acute and I know all three side lengths.

$$5.5^2 = 4.6^2 + 3.3^2 - 2(4.6)(3.3)\cos L$$
$$30.25 = 21.16 + 10.89 - 30.36\cos L$$
$$30.25 - 21.16 - 10.89 = -30.36\cos L$$
$$-1.8 = -30.36\cos L$$
$$\frac{-1.8}{-30.36} = \cos L$$
$$0.0593 \doteq \cos L$$
$$\cos^{-1}(0.0593) = \angle L$$
$$86.6^\circ \doteq \angle L$$

I can use the cosine law to solve for $\angle M$ because $\triangle LMN$ is acute and I know all three side lengths.

$$4.6^2 = 5.5^2 + 3.3^2 - 2(5.5)(3.3)\cos M$$
$$21.16 = 30.25 + 10.89 - 36.3\cos M$$
$$21.16 - 30.25 - 10.89 = -36.3\cos M$$
$$-19.98 = -36.3\cos M$$
$$\frac{-19.98}{-36.3} = \cos M$$
$$0.5504 \doteq \cos M$$
$$\cos^{-1}(-0.5504) = \angle M$$
$$56.6^\circ \doteq \angle M$$

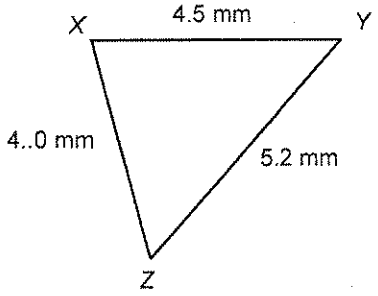
Now that I know $\angle L$ and $\angle M$, I can subtract the two known angles from 180° to determine $\angle N$.

$\angle N = 180^\circ - 56.6^\circ - 86.6$ (sum of angles in a triangle is 180°)

$\angle N = 36.8^\circ$

$\angle L$ is about 87°, $\angle M$ is about 57°, and $\angle N$ is about 37°.

d) Draw a triangle to model the problem.



I can use the cosine law to solve for $\angle X$ because $\triangle XYZ$ is acute and I know all three side lengths.

$$5.2^2 = 4.0^2 + 4.5^2 - 2(4.0)(4.5)\cos X$$
$$27.04 = 16.0 + 20.25 - 36\cos X$$
$$27.04 - 16.0 - 20.25 = -36\cos X$$
$$-9.21 = -36\cos X$$
$$\frac{-9.21}{-36} = \cos X$$
$$0.2558 \doteq \cos X$$
$$\cos^{-1}(0.2558) = \angle X$$
$$75.2^\circ \doteq \angle X$$

I can use the cosine law to solve for $\angle Y$ because $\triangle XYZ$ is acute and I know all three side lengths.

$$4.0^2 = 5.2^2 + 4.5^2 - 2(4.0)(4.5)\cos Y$$
$$16.0 = 27.04 + 20.25 - 46.8\cos Y$$
$$16.0 - 27.04 - 20.25 = -46.8\cos Y$$
$$-31.29 = -46.8\cos Y$$
$$\frac{-31.29}{-46.8} = \cos Y$$
$$0.6686 \doteq \cos Y$$
$$\cos^{-1}(0.6686) = \angle Y$$
$$48.0^\circ \doteq \angle Y$$

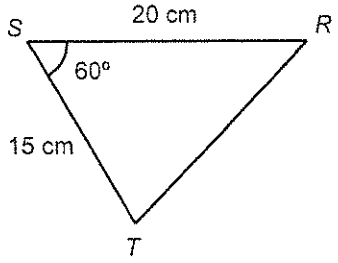
Now that I know $\angle X$ and $\angle Y$, I can subtract the two known angles from 180° to determine $\angle Z$.

$\angle Z = 180^\circ - 75.2^\circ - 48.0$ (sum of angles in a triangle is 180°)

$\angle Z = 56.8^\circ$

$\angle X$ is about 75°, $\angle Y$ is about 48°, and $\angle Z$ is about 57°.

6. Draw a triangle to model the problem.



I can use the cosine law to solve for s because $\triangle SRT$ is acute and I know two side lengths, r and t, and the angle between them, $\angle S$.

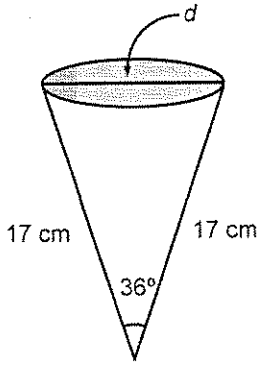
$$s^2 = 15^2 + 20^2 - 2(15)(20)\cos 60^\circ$$
$$s^2 = 325$$
$$s = \sqrt{325}$$
$$s \doteq 18$$

Now that I know all three side lengths (15 cm, 20 cm, and 18 cm), I can calculate the perimeter.

15 + 20 + 18 = 53

The perimeter of $\triangle SRT$ is about 53 cm.

7. Draw a diagram to model the problem.

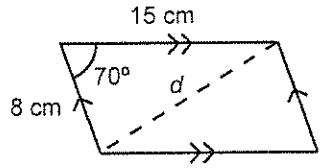


Let d represent the diameter of the cone. I can use the cosine law to solve for d because the triangle is acute and I know two side lengths, 17 cm and 17 cm, and the angle between them, 36°.

$$d^2 = 17^2 + 17^2 - 2(17)(17)\cos 36^\circ$$
$$d^2 \doteq 110.388\,177\,3$$
$$d \doteq \sqrt{110.388\,177\,3}$$
$$d \doteq 10.5$$

The top of the cone has a diameter of about 11 cm.

8. Draw a diagram to model the problem.



The diagonal divides the parallelogram into two acute triangles. Let d represent the diagonal. I can use the cosine law to solve for d because I know two side lengths, 8 cm and 15 cm, and the angle between them, 70°.

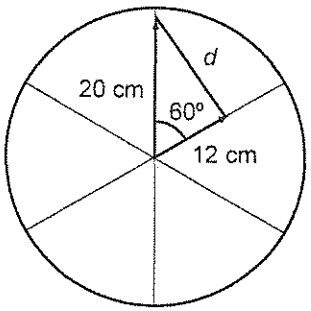
$d^2 = 8^2 + 15^2 - 2(8)(15)\cos 70^\circ$

9. I can use the cosine law to determine the angle because the triangle is acute and I know all three side lengths.

$$9.6^2 = 100.0^2 + 100.0^2 - 2(100.0)(100.0)\cos\theta$$
$$92.16 = 10\,000 + 10\,000 - 20\,000\cos\theta$$
$$92.16 - 10\,000 - 10\,000 = -20\,000\cos\theta$$
$$-19\,907.84 = -20\,000\cos\theta$$
$$\frac{-19\,907.84}{-20\,000} = \cos\theta$$
$$0.9954 \doteq \cos\theta$$
$$\cos^{-1}(0.9954) \doteq \theta$$
$$5.5° \doteq \theta$$

The pendulum swings through an angle of 5.5°.

10. a) Draw a diagram to model the problem.



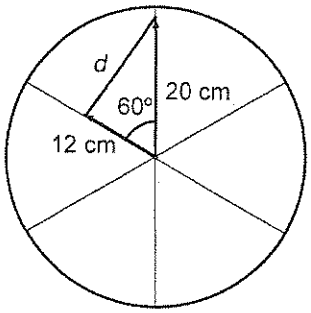
i) When it is 2:00, the angle between the minute hand and the hour hand is one sixth of the circle or 360° ÷ 6 = 60°.

Let d represent the distance between the tips of the hands. I can use the cosine law to determine d because the triangle is acute and I know two side lengths and the angle between them.

$$d^2 = 20^2 + 12^2 - 2(20)(12)\cos 60°$$
$$d^2 = 304$$
$$d = \sqrt{304}$$
$$d \doteq 17.4$$

The distance between the tips of the hands is about 17 cm.

ii) Draw a diagram to model the problem.



When it is 10:00, the angle between the minute hand and the hour hand is one sixth of the circle or 360° ÷ 6 = 60°.

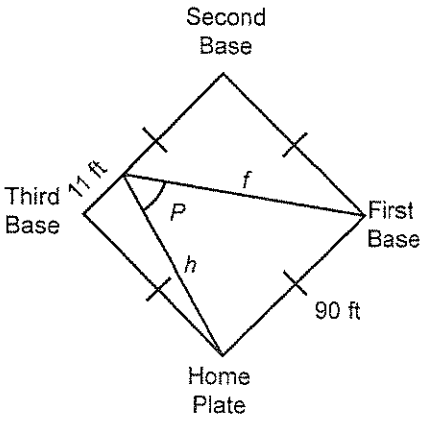
Let d represent the distance between the tips of the hands. I can use the cosine law to determine d because the triangle is acute and I know two side lengths and the angle between them.

$$d^2 = 20^2 + 12^2 - 2(20)(12)\cos 60°$$
$$d^2 = 304$$
$$d = \sqrt{304}$$
$$d \doteq 17.4$$

The distance between the tips of the hands is about 17 cm.

b) The answers are the same because the triangles formed by the hands at 2:00 and 10:00 are congruent.

11. Draw a diagram to model the problem. $\angle P$ is the angle between first base, the player, and home plate.



To solve for $\angle P$ using the cosine law, I can use the three side lengths of the triangle formed. I know one side is 90 ft, and I can determine f and h using the Pythagorean theorem.

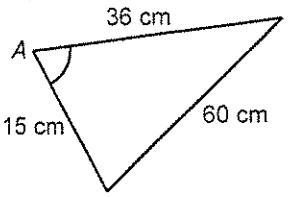
$$h^2 = 90^2 + 11^2 \qquad f^2 = 90^2 + (90 - 11)^2$$
$$h^2 = 8100 + 121 \qquad f^2 = 8100 + 6241$$
$$h = \sqrt{8221} \qquad f = \sqrt{14\,341}$$
$$h \doteq 90.7 \qquad f \doteq 119.8$$

Now I can use the cosine law to determine $\angle P$.

$$90^2 = 90.7^2 + 119.8^2 - 2(90.7)(119.8)\cos P$$
$$8100 = 8226.49 + 14\,352.04 - 21\,731.72\cos P$$
$$8100 - 8226.49 - 14\,352.04 = -21\,731.72\cos P$$
$$-14\,478.53 = -21\,731.72\cos P$$
$$\frac{-14\,478.53}{-21\,731.72} = \cos P$$
$$0.6662 \doteq \cos P$$
$$\cos^{-1}(0.6662) = \angle P$$
$$48° \doteq \angle P$$

The angle between first base, the player, and home plate is about 48°.

12.

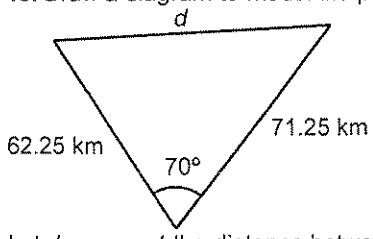


Since I knew all three side lengths, I tried using the cosine law to solve for one of the angles to see what happened.

$$60^2 = 36^2 + 15^2 - 2(36)(15)\cos A$$
$$3600 = 1296 + 225 - 1080\cos A$$
$$3600 - 1296 - 225 = -1080\cos A$$
$$2079 = -1080\cos A$$
$$\frac{-2079}{1080} = \cos A$$
$$-1.925 = \cos A$$
$$\cos^{-1}(-1.925) = \angle A$$

It looks like the angle opposite the 60 cm side has a negative cosine, which is impossible for an acute angle because my calculator shows that it is an error. That means that Sally cannot form an acute triangle with these pieces of lead.

13. Draw a diagram to model the problem.



Let d represent the distance between the drivers at 45 min. If I calculate the distance each driver has travelled, then I can use the cosine law to determine d because I will two side lengths and the angle between them.

45 min = 0.75 h

83.0 km/h × 0.75 h = 62.25 km

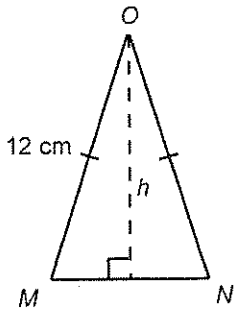
95.0 km/h × 0.75 h = 71.25 km

$$d^2 = 62.25^2 + 71.25^2 - 2(62.25)(71.25)\cos 70°$$
$$d^2 \doteq 5917.6926$$
$$d \doteq \sqrt{5917.6926}$$
$$d \doteq 76.9$$

The drivers are about 76.9 km apart.

14. Divide the decagon into 10 congruent isosceles triangles, each with two 12 cm sides. If I calculate the area of one, I can multiply by 10 to determine the total area.

I need to determine the base and height of one triangle to calculate its area.



If I determine the measure of two angles, I can use the sine law to determine the base, o, because I will know an opposite side-angle pair (n and $\angle N$), and another angle ($\angle O$).

Determine $\angle O$:

$\angle O = 360° \div 10 = 36°$

Determine $\angle N$:

Since $\triangle MNO$ is isosceles, $\angle N = \angle M$.

The sum of the angles in the triangle is 180°, so $\angle N$ or $\angle M = (180° - 36°) \div 2 = 72°$.

I can now use the sine law to determine the base.

$$\frac{12}{\sin 72°} = \frac{o}{\sin 36°}$$
$$\sin 36°\left(\frac{12}{\sin 72°}\right) = o$$
$$7.42 \doteq o$$

The base is about 7.42 cm.

Now that I know the base, I can use the Pythagorean theorem to determine the triangle's height, h.

$$12^2 = (0.5 \times 7.42)^2 + h^2$$
$$144 = 13.7507 + h^2$$
$$144 - 13.7507 = h^2$$
$$\sqrt{130.2493} = h$$
$$11.41 \doteq h$$

Calculate the area of one triangle.

$A = (0.5)(7.42)(11.41)$

$A \doteq 42.33$; the area is 42.33 cm^2

Calculate the area of the decagon.

$10(42.33) \doteq 423.3$

The decagon has an area of about 423 cm^2.

15. Answers will vary, e.g.,

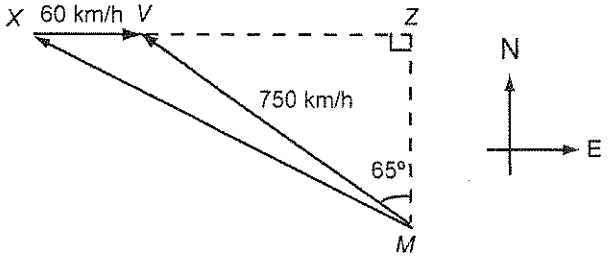
Problem:

Joe and Marie swim away from each other at an angle of 35°. Joe swims at 6 m/s and Marie swims at 7 m/s. How far apart are they after 5 s?

How to solve:

Determine the actual distance each swam using the speed and time (30 m and 35 m). You can then use cosine law to determine the distance between Joe and Marie, d, because you know two side lengths (30 m and 35 m) and the angle between them (35°).

16. a) Draw a diagram to model the problem.



I need to determine $\angle XMV$ and then add it to 65°.
If I determine $\angle XVM$, I can use the cosine law to determine XM.

$\angle ZVM = 180° - 90° - 65°$ (sum of angles in a triangle is 180°)
$= 25°$

$\angle XVM = 180° - 25°$ (supplementary angles)
$= 155°$

$XM^2 = 60^2 + 750^2 - 2(60)(750)\cos 155°$
$XM^2 \doteq 3600 + 562\,500 - (-81\,567.7008)$
$XM \doteq 804.8$

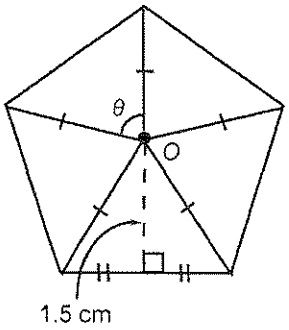
XM is about 805 km/h.
Now that I know XM, I can use the sine law to determine $\angle XMV$.

$$\frac{\sin XMV}{60} \doteq \frac{\sin 155°}{805}$$
$$\sin XMV \doteq \frac{60\sin 155°}{805}$$
$$\angle XMV \doteq \sin^{-1}\left(\frac{60\sin 155°}{809}\right)$$
$$\angle XMV \doteq 2°$$

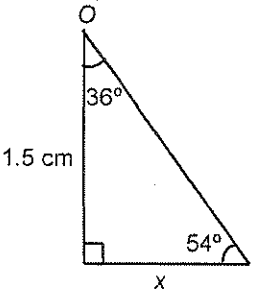
Compensation heading
$= 65° + \angle XVM$
$\doteq 65° + 2°$
$\doteq 67°$ west of north

b) The ground speed is represented by XM, which is about 805 km/h as calculated in part a).

17. Draw the pentagon divided into 5 congruent isosceles triangles, each with a height of 1.5 cm.



If I calculate the area of one triangle, I can multiply by 5 to determine the total area.
Since I know the height, I need to determine the base of one triangle to calculate its area. I can also use the base to calculate the perimeter.
Determine $\angle O$:
$\angle O = 360° \div 5 = 72°$
Determine the base angles.
Since the triangle is isosceles, the base angles are equal. The sum of the angles in the triangle is 180°, so either base angle $= (180° - 72°) \div 2 = 54°$.
Since the height of an isosceles triangle bisects the base, consider half the triangle in order to determine half the base, x.



Since it's a right triangle, I can use the tangent ratio to determine x.

$$\tan 36° = \frac{x}{1.5}$$
$$1.5(\tan 36°) = x$$
$$1.09 \doteq x$$

The base of the triangle is $2x$ or 2.2 cm.
Area $= 5(0.5)(2.18)(1.5)$
$= 8.2$
The area is about 8.2 cm^2.
Perimeter $= 5(2.18) = 10.9$
The perimeter is about 10.9 cm.

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1. a) Since two side lengths and the angle opposite one of them are known, you can use the sine law.
b) Since it's a right triangle and the opposite angle and adjacent side are known, you can use tangent ratio. You can also use the sine law, since you know two angles (90° and 47°) and a side.
c) Since three side lengths are known, you can use the cosine law.

2. a)
$$\frac{\sin\theta}{BC} = \frac{\sin B}{AC}$$
$$\frac{\sin\theta}{3.1} = \frac{\sin 60°}{2.7}$$
$$\sin\theta = \frac{3.1\sin 60°}{2.7}$$
$$\angle\theta = \sin^{-1}\left(\frac{3.1\sin 60°}{2.7}\right)$$
$$\angle\theta \doteq 84°$$

$\angle\theta$ is about 84°.

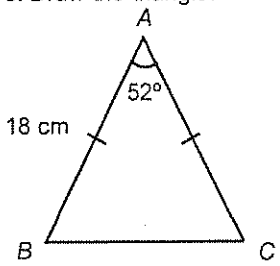
b)
$$\tan C = \frac{c}{b}$$
$$\tan 47° = \frac{c}{1.8}$$
$$1.8\tan 47° = c$$
$$1.9 \doteq c$$

Side c is about 1.9 cm.

c)
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$2.0^2 = 2.9^2 + 3.0^2 - 2(2.9)(3.0)\cos A$$
$$4.0 = 8.41 + 9.0 - 17.4\cos A$$
$$4.0 - 8.41 - 9.0 = -17.4\cos A$$
$$-13.41 = -17.4\cos A$$
$$0.7707 \doteq \cos A$$
$$\cos^{-1}(0.7707) = \angle A$$
$$40° \doteq \angle A$$

$\angle A$ is about 40°.

3. Draw the triangle.



a) Since the triangle is isosceles, the two base angles are equal.

$\angle B = (180° - 52°) \div 2$
$= 64°$
$\angle C = 64°$

The two equal angles in the triangle are 64°.

b) I can use the sine law to solve for a because I know two angles and a side length opposite one of them.

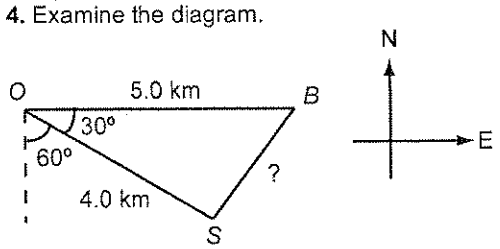
$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{a}{\sin 52°} = \frac{18}{\sin 64°}$$
$$a = \frac{18\sin 52°}{\sin 64°}$$
$$a \doteq 16$$

The length of the third side is about 16 cm.

c) Perimeter $\doteq$ 18 + 18 + 16 $\doteq$ 52.

The perimeter of the triangle is about 52 cm.

4. Examine the diagram.

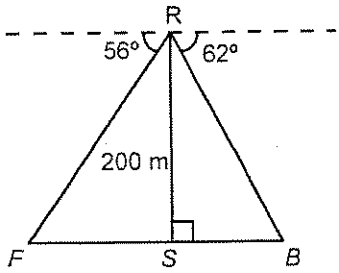


I can use the cosine law to solve for o because I know two side lengths and the angle between them.

$BS^2 = OB^2 + OS^2 - 2(OB)(OS)\cos O$
$BS^2 = 5.0^2 + 4.0^2 - 2(5.0)(4.0)\cos 30°$
$BS^2 \doteq 25.0 + 16.0 - 40(0.8660)$
$BS^2 \doteq 39.2679$
$BS \doteq 2.5$

The boats are about 2.5 km apart.

5. Draw a diagram to model the problem.



Use the angles of depression to determine $\angle RFS$ and $\angle RBS$.

$\angle RFS = 56°$ (alternate angles)
$\angle RBS = 62°$ (alternate angles)

I can use the tangent ratio of 56° to determine FS because I know the length of the side opposite to 56°.

$$\tan 56° = \frac{200}{FS}$$
$$FS(\tan 56°) = FS\left(\frac{200}{FS}\right)$$
$$FS(\tan 56°) = 200$$
$$FS = \frac{200}{\tan 56°}$$
$$FS \doteq 134.9$$

I can use the tangent ratio of 62° to determine SB because I know the length of the side opposite to 62°.

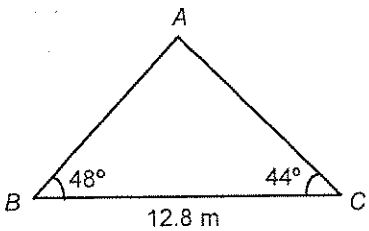
$$\tan 62° = \frac{200}{SB}$$
$$SB(\tan 62°) = SB\left(\frac{200}{SB}\right)$$
$$SB(\tan 62°) = 200$$
$$SB = \frac{200}{\tan 62°}$$
$$SB \doteq 106.3$$

Add FS and SB to determine the total length.

134.9 + 106.3 $\doteq$ 241 m

The sunken ship is about 241 m long.

6. Draw a diagram to model the problem.



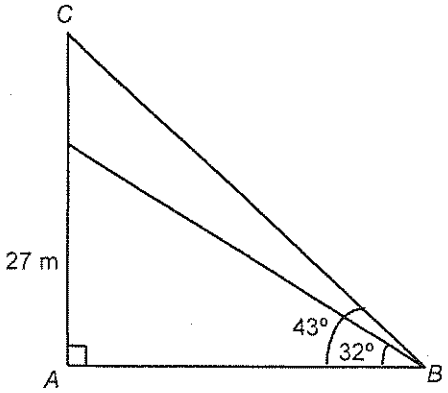
If I calculate $\angle A$, I can use the sine law to solve for c and b because I will know two angles and a side length opposite one of them.

$\angle A = 180° - 48° - 44°$ (sum of angles in a triangle)
$\angle A = 88°$

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$
$$\frac{c}{\sin 44°} = \frac{12.8}{\sin 88°}$$
$$c = \frac{12.8 \sin 44°}{\sin 88°}$$
$$c = 8.9$$
$$\frac{b}{\sin B} = \frac{a}{\sin A}$$
$$\frac{b}{\sin 48°} = \frac{12.8}{\sin 88°}$$
$$b = \frac{12.8 \sin 48°}{\sin 88°}$$
$$b = 9.5$$

The rafters are 8.5 m long and 9.5 m long.

7. Draw a diagram to model the problem.



a) I can use the tangent ratio of 32° to solve for AB because the triangle is a right triangle and I know the length of the side opposite 32°.

$$\tan 32° = \frac{27}{AB}$$
$$AB(\tan 32°) = AB\left(\frac{27}{AB}\right)$$
$$AB(\tan 32°) = 27$$
$$AB = \frac{27}{\tan 32°}$$
$$AB \doteq 43$$

The point on the ground is about 43 m away from the base of the building.

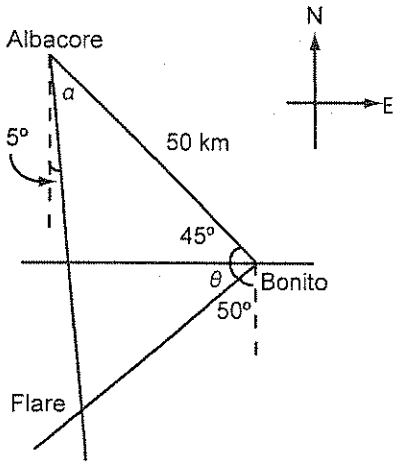
b) I can use the tangent ratio of 43° to solve for AC because $\triangle ABC$ is a right triangle and I know the length of the side adjacent to 43°.

$$\tan 43° = \frac{AC}{AB}$$
$$\tan 43° = \frac{AC}{43}$$
$$43 \tan 43° = AC$$
$$40 \quad AC$$

Subtract the height of the building to determine the height of the flag pole.

40 – 27 = 13 The flagpole is about 13 m tall.

8. Draw a diagram to model the problem.



If I calculate $\angle ABF$ and then $\angle AFB$, I can use the sine law to solve for BF and AF because I will know two angles and a side length opposite one of them.

$\theta = 90° - 50° = 40°$ (complementary angles)

$$\angle ABF = 45° + \theta$$
$$= 45° + 40°$$
$$= 85°$$

The two dashed lines are parallel so I can use the fact that co-interior angles have a sum of 180° to determine α.

$180° = (5° + \alpha) + (45° + \theta + 50°)$

$180° = 5° + \alpha + 45° + 40° + 50°$

$\alpha = 40°$

$$\angle AFB = 180° - \alpha - \angle ABF \text{ (sum of angles in a triangle)}$$
$$= 180° - 40° - 85°$$
$$= 55°$$

I can now use the sine law to determine BF.

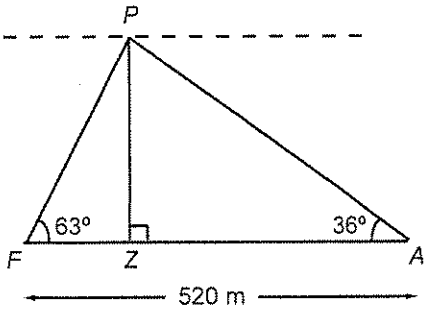
$$\frac{BF}{\sin \alpha} = \frac{AB}{\sin AFB}$$
$$\frac{BF}{\sin 40°} = \frac{50}{\sin 55°}$$
$$BF = \frac{50 \sin 40°}{\sin 55°}$$
$$BF \doteq 39$$

I can now use the sine law to determine AF.

$$\frac{AF}{\sin ABF} = \frac{AB}{\sin AFB}$$
$$\frac{AF}{\sin 85°} = \frac{50}{\sin 55°}$$
$$AF = \frac{50(\sin 85°)}{\sin 55°}$$
$$AF \doteq 61$$

The *Albacore* is about 61 km away and the Bonito is about 39 km away.

9. Draw a diagram to model the problem. Let P represent the plane, F represent Fred, and A represent Agnes.



If I calculate $\angle FPA$, I can use the sine law to solve for FP because I will know two angles and a side length opposite one of them.

$\angle FPA = 180° - 63° - 36°$

$\angle FPA = 81°$

$$\frac{FP}{\sin PAF} = \frac{AF}{\sin FPA}$$

$$\frac{FP}{\sin 36°} = \frac{520}{\sin 81°}$$

$$FP = \frac{520 \sin 36°}{\sin 81°}$$

$FP \doteq 309.5$

Now that I know FP, the hypotenuse of a right triangle, and one acute angle in the triangle, I can use the sine ratio to determine the side opposite to it.

$$\sin 63° = \frac{PZ}{FP}$$

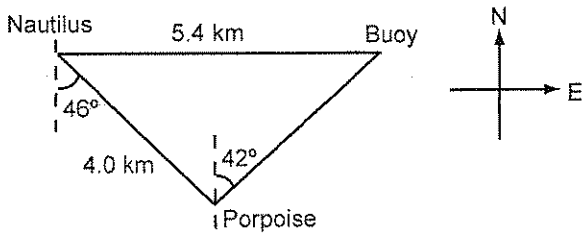
$PZ = FP \sin 63°$

$PZ \doteq 309.4583 \sin 63°$

$PZ \doteq 276$ m

The altitude is about 276 m.

10. Draw a diagram to model the problem.



If I calculate $\angle N$, I can use the cosine law to solve for n because I will know two side lengths and the angle between them.

$\angle N = 90° - 46°$

$= 44°$

$n^2 = p^2 + b^2 - 2\,pb \cos N$

$n^2 = 5.4^2 + 4.0^2 - 2(5.4)(4.0) \cos 44°$

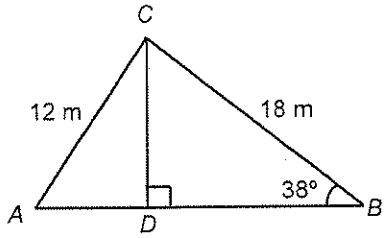
$n^2 = 29.16 + 16.0 - 43.2 \cos 44°$

$n^2 \doteq 14.0845$

$n \doteq 3.8$

The *Porpoise* is about 3.8 km from the buoy.

11. Draw a diagram to model the problem.



a) I can use the sine ratio of 38° to solve for CD because $\triangle BCD$ is a right triangle and I know the length of the hypotenuse.

$$\sin 38° = \frac{CD}{DB}$$

$$\sin 38° = \frac{CD}{18}$$

$18 \sin 38° = CD$

$11 \doteq CD$

The satellite dish tower is about 11 m tall.

b) Now that I know CD, I can use the Pythagorean theorem to calculate DB.

$CB^2 = CD^2 + DB^2$

$DB^2 = 18^2 - 11^2$

$DB^2 = 324 - 121$

$DB^2 = 203$

$DB = \sqrt{203}$

$DB \doteq 14.25$

Now that I know CD, I can use the Pythagorean theorem to calculate AD.

$CA^2 = CD^2 + AD^2$

$12^2 = 11^2 + AD^2$

$144 = 121 + AD^2$

$144 - 121 = AD^2$

$\sqrt{23} = AD$

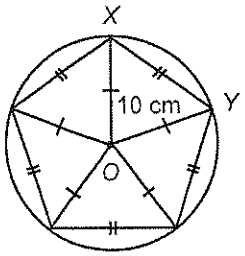
$4.88 = AD$

Add AD and DB to determine AB.

$4.88 + 14.25 = 19$

Points A and B are about 19 m apart.

12. Draw the diagram with the pentagon divided into five congruent isosceles triangles.



The perimeter is $5 \times XY$. If I calculate $\angle XOY$ and $\angle OXY$, I can use the sine law to solve for XY because I will know two angles and a side length opposite one of them.

$\angle XOY = 360° \div 5$ ($\angle XOY$ is $\frac{1}{5}$ of a circle)

$\angle XOY = 72°$

$\angle OXY = (180° - 72°) \div 2$ ($\triangle XOY$ is isosceles)

$\angle OXY = 54°$

$$\frac{XY}{\sin XOY} = \frac{OY}{\sin OXY}$$

$$\frac{XY}{\sin 72°} = \frac{10}{\sin 54°}$$

$$XY = \frac{10(\sin 72°)}{\sin 54°}$$

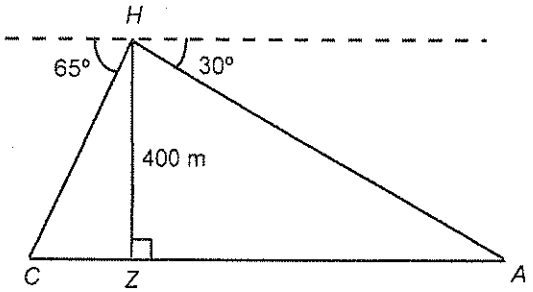
$$XY \doteq 11.78$$

Now I can determine the perimeter.

$$\begin{aligned} \text{Perimeter} &= 5(XY) \\ &\doteq 5(11.78) \\ &\doteq 59 \end{aligned}$$

The perimeter is about 59 cm.

13. a) Draw a diagram to model the problem. Let H represent the helicopter, C represent the car accident, and A represent the ambulance.



If I calculate $\angle HCZ$, I can use the tangent ratios of 65° to solve for CZ because $\triangle HCZ$ is a right triangle and I know the length of the side opposite $\angle HCZ$ (400 m).

$\angle HCZ = 65°$ (alternate angles)

$$\tan 65° = \frac{400}{CZ}$$

$$CZ(\tan 65°) = CZ\left(\frac{400}{CZ}\right)$$

$$CZ(\tan 65°) = 400$$

$$CZ = \frac{400}{\tan 65°}$$

$$CZ \doteq 186.52$$

If I calculate $\angle HAZ$, I can use the tangent ratios of 32° to solve for AZ because $\triangle HAZ$ is a right triangle and I know the length of the side opposite $\angle HAZ$ (400 m).

$\angle HAZ = 30°$ (alternate angles)

$$\tan 30° = \frac{400}{AZ}$$

$$AZ(\tan 30°) = AZ\left(\frac{400}{AZ}\right)$$

$$AZ(\tan 30°) = 400$$

$$AZ = \frac{400}{\tan 30°}$$

$$AZ \doteq 692.82$$

Calculate CA.

$$\begin{aligned} CA &= CZ + AZ \\ &\doteq 186.52 + 692.82 \\ &\doteq 879 \end{aligned}$$

The ambulance is about 879 m away.

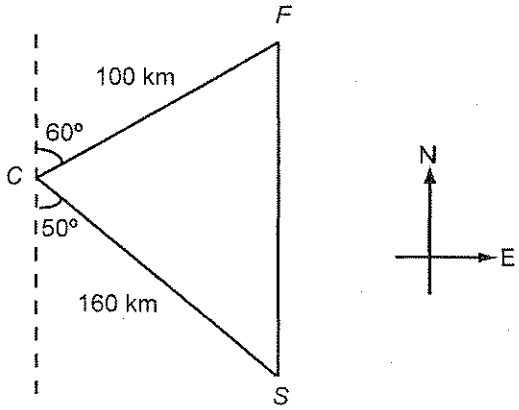
b) $\text{Time} = \dfrac{\text{distance}}{\text{speed}}$

$$\doteq \frac{0.879}{80} \qquad 879 \text{ m} = 0.879 \text{ km}$$

$$\doteq 0.01 \text{ h or } 40 \text{ s} \qquad 1 \text{ h} = 360 \text{ s}$$

The ambulance will take about 40 s.

14. a) Draw a diagram to model the problem. Let F represent one plane, S the other, and C the control tower.



If I calculate $\angle C$, I can use the cosine law to solve for c because I will know two side lengths and the angle between them.

$\angle C = 180° - 60° - 50°$ (supplementary angles)

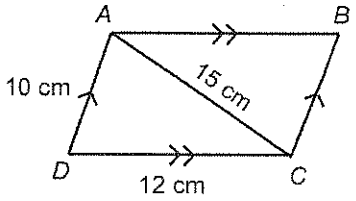
$\angle C = 70°$

$$\begin{aligned} c^2 &= s^2 + f^2 - 2\,sf\cos\angle C \\ c^2 &= 160^2 + 100^2 - 2(160)(100)\cos 70° \\ c^2 &= 25\,600 + 10\,000 - 32\,000\cos 70° \\ c^2 &\doteq 24\,655.3554 \\ c &\doteq 157 \end{aligned}$$

The airplanes are about 157 km apart.

b) Since both airplanes are flying at the same speed, the airplane closer to the control tower, 100 km, will reach it first.

15. Draw the parallelogram.



I can use the cosine law to solve for $\angle ADC$ because I know all three side lengths.

$$\begin{aligned} AC^2 &= AD^2 + CD^2 - 2\,(AD)(CD)\cos ADC \\ 15^2 &= 10^2 + 12^2 - 2(10)(12)\cos ADC \\ 225 &= 100 + 144 - 240\cos ADC \\ -19 &= -240\cos ADC \end{aligned}$$

$$\frac{-19}{-240} = \cos ADC$$

$$\begin{aligned} \angle ADC &= \cos^{-1}\left(\frac{19}{240}\right) \\ &\doteq 85° \end{aligned}$$

$\angle ADC = \angle ABC \doteq 85°$ (parallelogram)

I can use the cosine law to solve for $\angle DAC$ because I know all three side lengths.

$$CD^2 = AD^2 + AC^2 - 2(AD)(AC)\cos DAC$$
$$12^2 = 10^2 + 15^2 - 2(10)(15)\cos DAC$$
$$144 = 100 + 225 - 300\cos DAC$$
$$-181 = -300\cos DAC$$
$$\frac{-181}{-300} = \cos DAC$$
$$\angle DAC = \cos^{-1}\left(\frac{181}{300}\right)$$
$$\doteq 53°$$

Use $\angle ADC$ and $\angle DAC$ to determine $\angle DCA$.

$\angle DCA = 180° - \angle DAC - \angle ADC$ (sum of angles in a triangle)

$\angle DCA \doteq 180° - 53° - 85°$

$\angle DCA \doteq 42°$

Determine $\angle BAC$.

Since $\triangle ADC \cong \triangle CBA$, $\angle DCA = \angle BAC = 42°$, because they are corresponding angles.

Determine $\angle DAB$.

$$\angle DAB = \angle DAC + \angle BAC$$
$$\doteq 53° + 42°$$
$$\doteq 95°$$

$\angle DCB = \angle DAB = 95°$ (parallelogram)

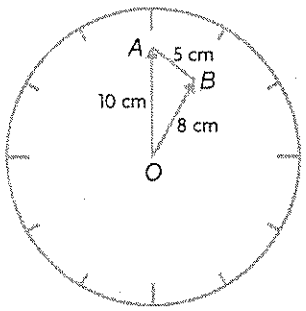
The angles are about 85°, 95°, 85°, and 95°.

16. Answers will vary. e.g.,

Problem: The minute hand of a clock is pointing at the number 12 and is 10 cm long. The hour hand is 8 cm long. The distance between the tips of the hands is 5 cm. What time could it be?

Solution:

Draw a diagram to model the problem.



I can use the cosine law to solve for $\angle O$ because I know all three side lengths.

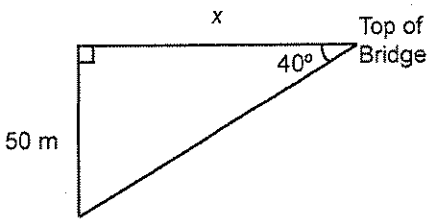
$$o^2 = b^2 + a^2 - 2\,ba\cos O$$
$$5^2 = 10^2 + 8^2 - 2(10)(8)\cos O$$
$$25 = 100 + 64 - 160\cos O$$
$$-139 = -160\cos O$$
$$\frac{-139}{-160} = \cos O$$
$$\angle O = \cos^{-1}\left(\frac{139}{160}\right)$$
$$\angle O \doteq 30°$$

A 30° angle is $\frac{1}{12}$ of the circular clock face

360° ÷ 30° = 12, which means the hands are one number apart.

There are two possible times depending on where the hour hand is positioned, behind or ahead of the minute hand, 1 o'clock or 11 o'clock.

17. Let x represent the distance along the water between the bridge and boat 1. Draw a diagram to model the situation for Boat 1.



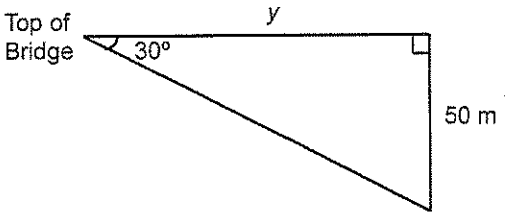
Boat 1

I can use the tangent ratio of 40° to solve for x because I know the length of the side opposite 40°.

$$\tan 40° = \frac{50}{x}$$
$$x(\tan 40°) = x\left(\frac{50}{x}\right)$$
$$x(\tan 40°) = 50$$
$$x = \frac{50}{\tan 40°}$$
$$x \doteq 59.5877$$

Boat 1 is about 59.5877 m away from the bridge.

Let y represent the distance between the bridge and boat 2. Draw a diagram to model the situation for Boat 2.



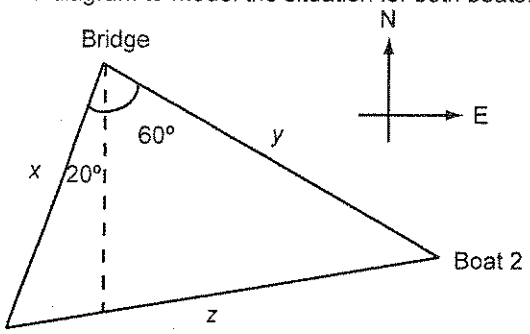
Boat 2

I can use the tangent ratio of 30° to solve for y because I know the length of the side opposite 30°.

$$\tan 30° = \frac{50}{y}$$
$$y(\tan 30°) = y\left(\frac{50}{y}\right)$$
$$y(\tan 30°) = 50$$
$$y = \frac{50}{\tan 30°}$$
$$y \doteq 86.6025$$

The second boat is about 86.6025 m away from the bridge.

Let z represent the distance between the two boats.
Draw a diagram to model the situation for both boats.



Boat 1

$z^2 = x^2 + y^2 - 2xy\cos 80°$

$z^2 = (59.5877)^2 + (86.6025)^2 - 2(59.5877)(86.6025)\cos 80°$

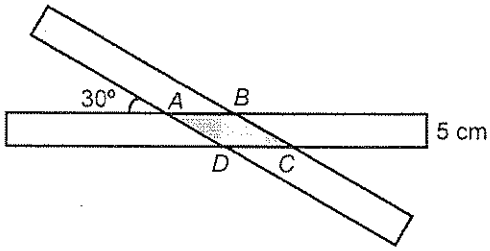
$z^2 \doteq 3550.7 + 7500.0 - 1792.2$

$z = \sqrt{9258.5}$

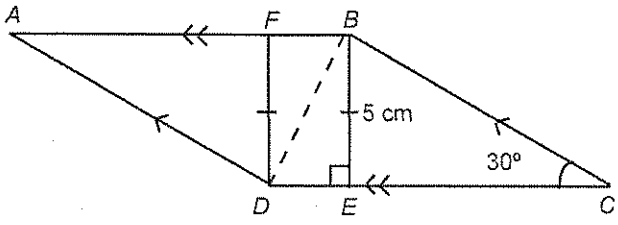
$z \doteq 96.2$

The two boats are about 96 m apart.

18. Draw and label a diagram to model the problem.



$ABCD$ is a parallelogram with $AB = CD$ and $AD = BC$.
To determine its area I need to know its height and base.
The height is 5 cm, the width of the horizontal strip.
I need to determine the base length, DC.
$\angle A = 30° = \angle C$ (opposite angles)
Draw a diagram to model the problem.



I can use the sine ratio of 30° to solve for BC because $\triangle BCE$ is a right triangle and I know the length of the opposite side.

$$\sin 30° = \frac{5}{BC}$$

$$BC(\sin 30°) = BC\left(\frac{5}{BC}\right)$$

$$BC(\sin 30°) = 5$$

$$BC = \frac{5}{\sin 30°}$$

$$BC = 10 \text{ cm}$$

BC is 10 cm.

Now that I know BC, I can use the Pythagorean theorem to determine EC.

$BC^2 = BE^2 + EC^2$

$10^2 = 5^2 + EC^2$

$EC^2 = 100 - 25$

$EC^2 = 75$

$EC \doteq 8.6603$

EC is about 8.7 cm.

Now I need to determine DE.

If I calculate $\angle DBE$, I can use the tangent ratio to solve for DE because $\triangle DBE$ is a right triangle and I know the length of the side adjacent to $\angle DBE$, 5 cm.

$\angle CBE = 180° - 90° - 30°$ (sum of angles in a triangle)

$\angle CBE = 60°$

$\angle ABC = \angle ABE + 60°$

$\angle ABC = 90° + 60°$

$\angle ABC = 150°$

DB bisects $\angle ABC$, so $\angle ABD = 75°$ and $\angle DBC = 75°$.

$\angle DBC = \angle DBE + \angle CBE$

$75° = \angle DBE + 60°$

$\angle DBE = 15°$

$$\tan 15° = \frac{DE}{5}$$

$5(\tan 15°) = DE$

$1.3397 \doteq DE$

DE is about 1.3 cm.

Calculate the length of the base.

$DC = DE + EC$

$\doteq 1.3 + 8.7$

$\doteq 10.0$

DC is about 10 cm.

Calculate the area of $ABCD$.

Area of parallelogram $= bh$

$\doteq 10.0 \times 5$

$\doteq 50.0$

The area of the overlap is 50.0 cm^2.

Chapter Review, page 453

1. No. I can tell $\triangle ABC$ is a right triangle from the given information: $6^2 + 8^2 = 10^2$, so $\triangle ABC$ is a right triangle.

2. a) $\frac{d}{\sin D} = \frac{e}{\sin E} = \frac{f}{\sin F}$, so part a) is correct.

b) $\frac{\sin D}{d} = \frac{\sin E}{e} = \frac{\sin F}{f}$, so part b) is correct.

c) $\frac{\sin E}{e} = \frac{\sin F}{f}$, so part c) is correct.

$f \sin E = e \sin F$

d) $\frac{\sin D}{d} = \frac{\sin F}{f}$, so part d) is not correct.

3. a) If I calculate $\angle X$ (the angle opposite x), I can use the sine law to solve for x because I will know two angles and a side length opposite one of them.

$\angle X = 180° - 67° - 58° = 55°$

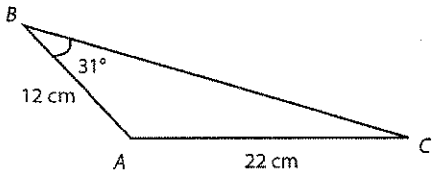
$$\frac{x}{\sin X} = \frac{24.5}{\sin 58°}$$
$$\frac{x}{\sin 55°} = \frac{24.5}{\sin 58°}$$
$$x = \frac{24.5 \sin 55°}{\sin 58°}$$
$$x \doteq 23.7$$

The length of x is about 23.7 m.

b) I can use the sine law to solve for θ because I know two angles and a side length opposite one of them.

$$\frac{\sin\theta}{35.4} = \frac{\sin 51°}{31.2}$$
$$\sin\theta = \frac{35.4 \sin 51°}{31.2}$$
$$\theta = \sin^{-1}\left(\frac{35.4 \sin 51°}{31.2}\right)$$
$$\theta \doteq 62°$$

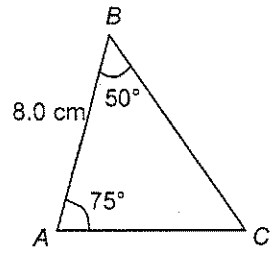
4. Draw the triangle.



I can use the sine law to solve for $\angle C$ because I know two angles and a side length opposite one of them.

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$
$$\frac{\sin C}{12} = \frac{\sin 31°}{22}$$
$$\sin C = \frac{12 \sin 31°}{22}$$
$$\angle C = \sin^{-1}\left(\frac{12 \sin 31°}{22}\right)$$
$$\angle C \doteq 16°$$

5. Draw the triangle.



If I calculate $\angle C$, I can use the sine law to solve for both a and b because I will know two angles and a side length opposite one of them.

$\angle C = 180° - 75° - 50°$ (sum of angles in a triangle
$= = 55°$ is 180°)

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{a}{\sin 75°} = \frac{8.0}{\sin 55°}$$
$$a = \frac{8.0(\sin 75°)}{\sin 55°}$$
$$a \doteq 9.4$$

Side a is about 9.4 cm.

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\frac{b}{\sin 50°} = \frac{8.0}{\sin 55°}$$
$$b = \frac{8.0(\sin 50°)}{\sin 55°}$$
$$b \doteq 7.5$$

Side b is about 7.5 cm.

6. If I calculate $\angle K$ (the angle at the kite), I can use the sine law to solve for k (the distance between Allison and Marc) because I will know two angles and a side length opposite one of them.

$\angle K = 180° - 70° - 30°$ (sum of angles in a triangle
$\angle K = 80°$ is 180°)

$$\frac{k}{\sin K} = \frac{m}{\sin M}$$
$$\frac{k}{\sin 80°} = \frac{150}{\sin 30°}$$
$$k = \frac{150(\sin 80°)}{\sin 30°}$$
$$k \doteq 295$$

Marc is about 295 m from Allison.

7. a) This is not a form of the cosine law because it should end with cos A instead of cos B:
$a^2 = b^2 + c^2 - 2bc \cos \underline{A}$
The side length on the LS of the equation, a, should be opposite the angle at the end of the RS of the equation for which the cosine is used, but it's not. In this case it is $\angle C$ and should be $\angle A$.

b) This is a form of the cosine law because the side length on the LS of the equation, c, is opposite the angle at the end of the RS of the equation, for which the cosine is used, $\angle C$.

c) This is a form of the cosine law because the side length on the LS of the equation, b, is opposite the angle at the end of the RS of the equation, for which the cosine is used, $\angle B$.

8. a) I can use the cosine law to solve for x because I know two side lengths and the angle between them.

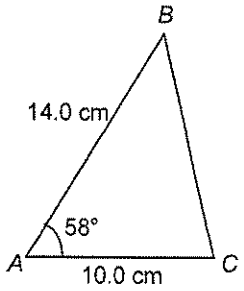
$$x^2 = 5.0^2 + 6.0^2 - 2(5.0)(6.0)\cos 87°$$
$$x^2 = 25 + 36 - 60\cos 87°$$
$$x^2 \doteq 57.86$$
$$x = \sqrt{57.86}$$
$$x \doteq 7.6$$

Side x is about 7.6 cm.

b) I can use the cosine law to solve for θ because I know all three side lengths.

$$14.0^2 = 15.0^2 + 7.0^2 - 2(15.0)(7.0)\cos\theta$$
$$196 = 225 + 49 - 210\cos\theta$$
$$196 - 225 - 49 = -210\cos\theta$$
$$-78 = -210\cos\theta$$
$$\frac{-78}{-210} = \cos\theta$$
$$0.3714 \doteq \cos\theta$$
$$\cos^{-1}(0.3714) = \angle\theta$$
$$68° \doteq \angle\theta$$

9. Draw the triangle.



I can use the cosine law to solve for a because I know two side lengths and the angle between them.

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$a^2 = 10.0^2 + 14.0^2 - 2(10.0)(14.0)\cos 58°$$
$$a^2 = 100 + 196 - 280\cos 58°$$
$$a^2 \doteq 147.623$$
$$a = \sqrt{147.623}$$
$$a \doteq 12.2$$

Side a is about 12.2 cm.

I can use the sine law to solve for $\angle B$ because I know two sides and the angle opposite one of them.

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$
$$\frac{\sin B}{10.0} = \frac{\sin 58°}{12.2}$$
$$\sin B = \frac{10.0(\sin 58°)}{12.2}$$
$$\sin B \doteq 0.6951$$
$$\angle B = \sin^{-1}(0.6951)$$
$$\angle B \doteq 44°$$

I can calculate $\angle C$ now because I know the other two angles in the triangle.

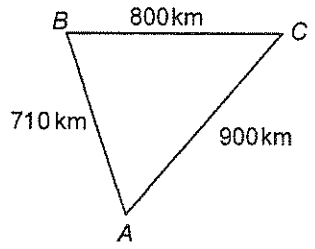
$\angle C = 180° - 58° - 44°$ (sum of angles in a triangle)

$\angle C \doteq 78°$ (since $\angle B \doteq 78°$)

10. The first airplane travels $2 \times 355 = 710$ km in 2 h.

The second airplane travels $2 \times 450 = 900$ km in 2 h.

Draw a diagram to model the problem.

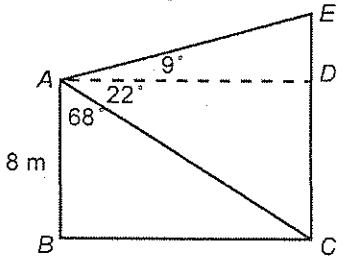


I can use the cosine law to solve for $\angle A$ because I know all three side lengths.

$$800^2 = 710^2 + 900^2 - 2(710)(900)\cos A$$
$$640\ 000 = 504\ 100 + 810\ 000 - 1\ 278\ 000\cos A$$
$$640\ 000 - 504\ 100 - 810\ 000 = -1\ 278\ 000\cos A$$
$$-674\ 100 = -1\ 278\ 000\cos A$$
$$0.5275 \doteq \cos A$$
$$\cos^{-1}(0.5275) = \angle A$$
$$58° \doteq \angle A$$

The angle between the paths of the airplanes is about 58°.

11. Draw a diagram to model the problem.



I need to determine the length of CE, which is $DE + CD$.

$CD = 8$ m because $CD = AB = 8$ m.

If I determine the length of BC, I will know AD because $BC = AD$.

Since $\angle DAB$ is a right angle, $\angle CAD$ and $\angle CAB$ are complementary angles.

So $\angle CAB = 90° - 22° = 68°$

I can use the cosine ratio of 68° to solve for AC because $\triangle ABC$ is a right triangle and I know the length of the side adjacent to 68°.

$$\cos 68° = \frac{8}{BC}$$
$$BC(\cos 68°) = BC\left(\frac{8}{BC}\right)$$
$$\frac{BC(\cos 68°)}{\cos 68°} = \frac{8}{\cos 68°}$$
$$BC = \frac{8}{\cos 68°}$$
$$BC \doteq 21.4$$

Since CE and AB are both parallel, $\angle ECA = \angle BAC$ because they are alternate angles. Using the fact that the sum of the interior angles of a triangle must be 180°, $\angle CEA = 180° - 31° - 68° = 81°$.

Using the sine law

$$\frac{EC}{\sin 31°} \doteq \frac{21.4}{\sin 81°}$$

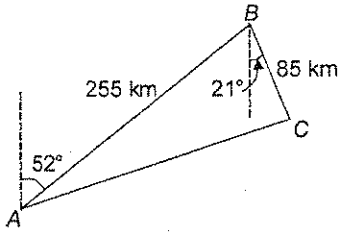
$$\sin 31°\left(\frac{EC}{\sin 31°}\right) \doteq \sin 31°\left(\frac{21.4}{\sin 81°}\right)$$

$$EC \doteq \frac{21.4 \sin 31°}{\sin 81°}$$

$$EC \doteq 11$$

The height of the pole is about 11 m.

12. Draw a diagram to model the problem.



I need to know the perimeter of $\triangle ABC$.

$P = a + b + c = 85 \text{ km} + b + 255 \text{ km}$.

If I calculate $\angle B$, I can use the cosine law to solve for b because I will know two side lengths and the angle between them.

Since the dashed lines are parallel, I know that

$\angle B = 52° + 21° = 73°$ (52° alternate angles)

$b^2 = 255^2 + 85^2 - 2(255)(85)\cos 73°$

$b^2 = 65\ 025 + 7225 - 43\ 350\cos 73°$

$b^2 \doteq 59\ 757.687$

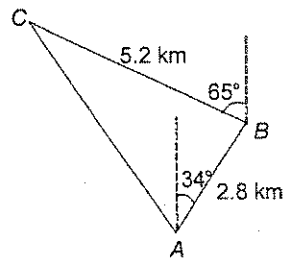
$b = \sqrt{59\ 757.687}$

$b \doteq 244$

$P = 85 + 244 + 255 = 584$

The total distance is about 584 km.

13. Draw a diagram to model the problem.



I need to know side length b and the other part of $\angle A$.

If I calculate $\angle B$, I can use the cosine law to solve for b because I will know two side lengths and the angle between them.

Since the dashed lines are parallel, I know that there are 34° alternate angles and $34° + \angle B + 65°$ are supplementary angles.

So, $\angle B = 180° - 34° - 65° = 81°$.

$b^2 = 2.8^2 + 5.2^2 - 2(2.8)(5.2)\cos 81°$

$b^2 = 7.84 + 27.04 - 29.12\cos 81°$

$b^2 \doteq 30.3246$

$b = \sqrt{30.3246}$

$b \doteq 5.5$

I can use the sine law to solve for $\angle A$ because I know two sides and the angle opposite one of them.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin A}{5.2} = \frac{\sin 81°}{5.5}$$

$$\sin A = \frac{5.2(\sin 81°)}{5.5}$$

$$\sin A \doteq 0.9338$$

$$\angle A = \sin^{-1}(0.9338)$$

$$\angle A \doteq 69°$$

The other part of $\angle A$ is $69° - 34° = 35°$.

The direction is 35° west of north.

The second canoeist must paddle about 5.5 km N35°W.

Chapter Self-Test, pg 454

1. a) I can use the sine law to solve for $\angle B$ or θ because I know two sides and the angle opposite one of them.

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin \theta}{4.1} = \frac{\sin 82°}{6.0}$$

$$\sin \theta = \frac{4.1(\sin 82°)}{6.0}$$

$$\sin \theta \doteq 0.6767$$

$$\angle \theta = \sin^{-1}(0.6767)$$

$$\angle \theta \doteq 43°$$

b) If I calculate $\angle C$, I can use the sine law to solve for c because I will know two angles and a side length opposite one of them.

$\angle C = 180° - 78° - 69°$ (sum of angles in a triangle is 180°)

$\angle C = 33°$

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

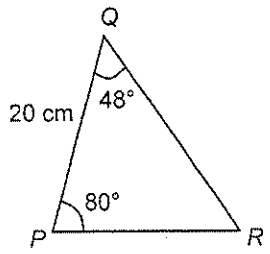
$$\frac{c}{\sin 33°} = \frac{4.25}{\sin 78°}$$

$$c = \frac{4.25(\sin 33°)}{\sin 78°}$$

$$c \doteq 2.37$$

Side c is about 2.37 cm.

2. Draw the triangle.



If I calculate $\angle R$, I can use the sine law to solve for q and for p because I will know two angles and a side length opposite one of them.

$\angle R = 180° - 80° - 48°$ (sum of angles in a triangle)
$\angle R = 52°$

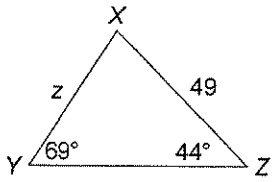
$$\frac{q}{\sin Q} = \frac{r}{\sin R}$$
$$\frac{q}{\sin 48°} = \frac{20}{\sin 52°}$$
$$q = \frac{20(\sin 48°)}{\sin 52°}$$
$$q \doteq 19$$

Side q is about 19 cm.

$$\frac{p}{\sin P} = \frac{r}{\sin R}$$
$$\frac{p}{\sin 80°} = \frac{20}{\sin 52°}$$
$$p = \frac{20(\sin 80°)}{\sin 52°}$$
$$p \doteq 25$$

Side p is about 25 cm.

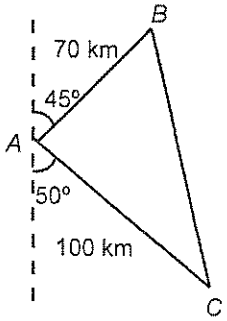
3. a) Answers will vary, e.g., I drew a triangle and included two angles and a side length opposite one of them so that I could use the sine law.



b) $$\frac{z}{\sin Z} = \frac{y}{\sin Y}$$
$$\frac{z}{\sin 44°} = \frac{49}{\sin 69°}$$
$$z = \frac{49(\sin 44°)}{\sin 69°}$$
$$z \doteq 36$$

Side z is about 36 units.

4. Draw a diagram to model the problem



If I calculate $\angle A$, I can use the cosine law to solve for a because I will know two side lengths and the angle between them.

$\angle A = 180° - 45° - 50° = 85°$ (supplementary angles)
$a^2 = 70^2 + 100^2 - 2(70)(100)\cos 85°$
$a^2 = 4900 + 10\,000 - 14\,000\cos 85°$
$a^2 \doteq 13\,680$
$a \doteq \sqrt{13\,680}$
$a \doteq 117$

The boats are about 117 km apart.

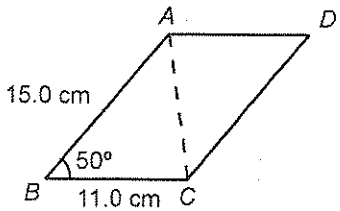
5. If I calculate $\angle A$, I can use the cosine law to solve for c because I will know two side lengths and the angle between them.

$\angle A = 180° - 72° - 53°$ (sum of angles in a triangle is 180°)
$\angle A = 55°$

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$
$$\frac{c}{\sin 53°} = \frac{515.0}{\sin 55°}$$
$$c = \frac{515.0(\sin 53°)}{\sin 55°}$$
$$c \doteq 502.1$$

The bridge is about 502.1 m long.

6. Draw the parallelogram.

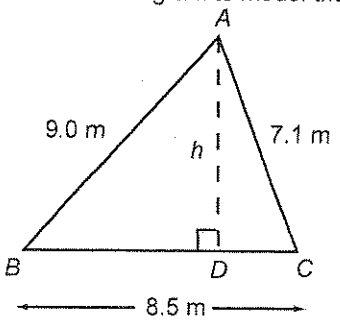


I can use the cosine law to solve for AC because I know two side lengths and the angle between them.

$AC^2 = 11.0^2 + 15.0^2 - 2(11.0)(15.0)\cos 50°$
$AC^2 = 121 + 225 - 330\cos 50°$
$AC^2 \doteq 133.88$
$AC = \sqrt{133.88}$
$AC \doteq 11.6$

The length of the shorter diagonal is about 11.6 cm.

7. Draw a diagram to model the problem.



To determine the area, I need to know the base and the height. The base is 8.5 m. If I calculate $\angle B$, I can use the sine ratio of $\angle B$ to determine the height, h, because $\triangle ABD$ is a right triangle and I know the length of the hypotenuse.

I can use the cosine law to solve for $\angle B$ because I know all three side lengths.

$$7.1^2 = 8.5^2 + 9.0^2 - 2(8.5)(9.0)\cos B$$
$$50.41 = 72.25 + 81 - 153\cos B$$
$$153\cos B = 72.25 + 81 - 50.41$$
$$\cos B = \frac{102.84}{153}$$
$$\angle B = \cos^{-1}\left(\frac{102.84}{153}\right)$$
$$\angle B \doteq 47.7°$$

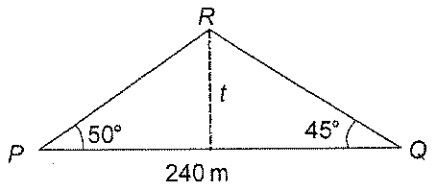
$$\sin 47.7° = \frac{h}{9.0}$$
$$9.0\sin 47.7° = h$$
$$6.66 \doteq h$$

Calculate the area.

$$\text{Area} = \frac{1}{2}bh$$
$$\doteq \frac{1}{2}(8.5)(6.66)$$
$$\doteq 28.3$$

The area of the patio is about 28.3 m^2.

8. Draw a diagram to model the problem.



If I calculate $\angle R$, I can use the sine law to solve for q because I will know two angles and a side length opposite one of them.

$\angle R = 180° - 50° - 45°$ (sum of angles in a triangle)

$\angle R = 85°$

$$\frac{q}{\sin Q} = \frac{r}{\sin R}$$
$$\frac{q}{\sin 45°} = \frac{240}{\sin 85°}$$
$$q = \frac{240(\sin 45°)}{\sin 85°}$$
$$q \doteq 170.4$$

Now I can use the sine ratio of 50° to solve for t because I now know the length of the hypotenuse, 170 m.

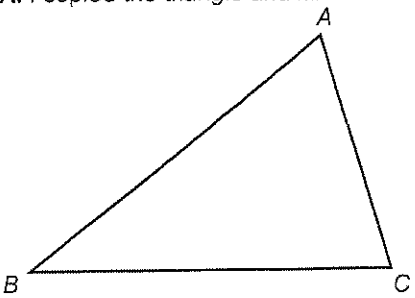
$$\sin 50° = \frac{t}{170.4}$$
$$170.4\sin 50° = t$$
$$130 \doteq t$$

The height of the tower is about 131 m.

9. Answers may vary, e.g., if the 37° angle is formed by the two given sides, you can use the cosine law to determine the third side. If the 37° angle is not formed by the two given sides, you can use the sine law to determine a second angle, use the fact that the sum of the angles in a triangle is 180° to determine the third angle, and then use the sine law again to determine the third side.

Chapter Task, page 455

A. I copied the triangle and labelled the vertices.



I determined the actual lengths of the sides by measuring each side and using the scale 1.5 cm : 20 km.
The lengths of the sides on the map are
$BC \doteq 5.3$ cm, $AC \doteq 3.6$ cm, and $AB \doteq 5.6$ cm.
So the actual side lengths are

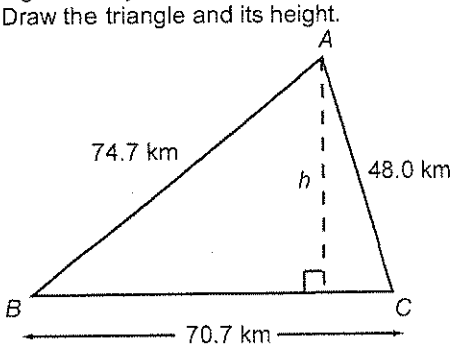
$$BC \doteq \frac{5.3}{1.5} \times 20 \doteq 70.7 \text{ km}$$
$$AC \doteq \frac{3.6}{1.5} \times 20 \doteq 48.0 \text{ km}$$
$$AB \doteq \frac{5.6}{1.5} \times 20 \doteq 74.7 \text{ km}$$

B. Answers may vary, e.g.,
To determine the area, I needed the base length and the height. I knew the base, BC, was 70.7 km, from part A, so I measured the height and then used the scale:
The height of the triangle on the map is about 3.4 cm.
The actual height is

$$\frac{3.4}{1.5} \times 20 \doteq 45.3 \text{ km}.$$

Area = $0.5 \times BC \times h$
Area $\doteq$ (0.5)(70.7)(45.3)
Area $\doteq$ 1601.4
The actual area is about 1601.4 km^2.

C. I knew the base length, 70.7 km from part A, so I needed to figure out the height of the triangle using trigonometry, so I could calculate the area.
Draw the triangle and its height.



If I calculate $\angle B$, I can use its sine ratio to solve for h because I know the hypotenuse, 74.7 km.
I can use the cosine law to solve for $\angle B$ because I know all three side lengths.

$$AC^2 = BC^2 + AB^2 - 2(BC)(AB)\cos B$$
$$48.0^2 = 70.7^2 + 74.7^2 - 2(70.7)(74.7)\cos B$$
$$2304 = 4998.49 + 5580.09 - 10\,562.58\cos B$$

$$2304 - 4998.49 - 5580.09 = -10\,562.58\cos B$$
$$-8274.58 = -10\,562.58\cos B$$
$$0.7834 \doteq \cos B$$
$$\cos^{-1}(0.7834) = \angle B$$
$$38.4^\circ \doteq \angle B$$

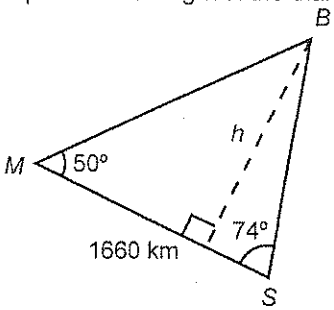
Then I can use the sine ratio to determine h:

$$\sin B = \frac{h}{AB}$$
$$\sin 38.4^\circ = \frac{h}{74.7}$$
$$74.7\sin 38.4^\circ = h$$
$$46.4 \doteq h$$

The height of the triangle is about 46.4 km.
Calculate the area.

$$\text{Area} = \frac{BC \times h}{2}$$
$$\doteq \frac{70.7 \times 46.4}{2}$$
$$\doteq 1640.2$$

The area of the vortex is about 1640.2 km^2.
D. The answers for part B and C are fairly close.
E. To determine the area, I need to know the base and the height. The base, MS, is given, 1660 km.
Draw a diagram to model the problem. Let B represent Bermuda, M represent Miami, and S represent San Juan, Puerto Rico. Draw a perpendicular from B to MS to represent the height of the triangle.



If I calculate $\angle MBS$, I can use the sine law to solve for SB because I will know two angles and a side length opposite one of them.
$\angle MBS = 180^\circ - 50^\circ - 74^\circ$ (sum of angles in a triangle)
$\angle MBS = 56^\circ$

$$\frac{1660}{\sin 56^\circ} = \frac{SB}{\sin 50^\circ}$$
$$\sin 50^\circ\left(\frac{1660}{\sin 56^\circ}\right) = SB$$
$$2613.8 \doteq SB$$

Side SB is about 2613.8 km.

Now that I know SB, I can use the sine ratio of 74° to solve for h because I know the hypotenuse.

$$\sin S = \frac{h}{BS}$$
$$\sin 74^\circ \doteq \frac{h}{2613.8}$$
$$2613.8\sin 74^\circ = h$$
$$2512.5 \doteq h$$

The height of the triangle is about 2512.5 km.
Calculate the area.

$$\text{Area} = \frac{MS \times h}{2}$$
$$\doteq \frac{1660 \times 2512.5}{2}$$
$$\doteq 2\,085\,413.0$$

The area of the Bermuda Triangle is about 2 085 413.0 km^2.
F. I figured out the rate per square kilometre of ship wrecks for each to see which was more dangerous.
Marysburgh Vortex

$$\frac{450}{1640.2} \doteq 0.27$$

The rate of the Marysburgh Vortex is about 0.27 wrecks/km^2.
Bermuda Triangle

$$\frac{1000}{2\,085\,413.0} \doteq 0.0005$$

The rate of the Bermuda Triangle is about 0.0005 wrecks/km^2.
The Marysburgh Vortex is more dangerous.

Cumulative Review Chapters 7–8, pages 456

1. B
A. $\angle F = 180^\circ - 45^\circ - 62^\circ$
$= 73^\circ$
Not similar because the angles are not congruent.
B. Since the sides from longest to shortest are f, e, d, check if $f{:}c = e{:}b = d{:}a$.
$f{:}c = 12.5{:}5.0$
$= 2.5$
$e{:}b = 10.5{:}4.2$
$= 2.5$
$d{:}a = 9.0{:}3.6$
$= 2.5$
Similar because all sides are proportional.
C. $\angle F = \angle C$
Since e is longer than d, check if $e{:}b = d{:}a$.
$d{:}a = 7.2{:}3.6$
$= 2.0$
$e{:}b = 12.6{:}4.2$
$= 3$
Not similar because sides are not proportional.
D. $\angle E = \angle B$
Since d is longer than f, check if $d{:}c = f{:}a$.
$d{:}c = 10.8{:}5.0$
$= 2.16$
$f{:}a = 10{:}3.6$
$\doteq 2.8$
Not similar because sides are not proportional.

2. B

Determine the length of the hypotenuse of the scale diagram, x.

$x = \sqrt{7^2 + 7^2}$

$x = \sqrt{98}$

$x \doteq 9.9$

The length of the hypotenuse is about 9.9 cm or 0.099 m. The scale is about $\frac{3}{0.099} \doteq 30.3$.

Determine the lengths, y, of the perpendicular sides in the real garden.

$y \doteq 30.3 \times 7$

$y \doteq 212.1$

The lengths of the sides in the real garden are about 212.1 cm or 2.121 m.

$\text{Area} \doteq \frac{1}{2}(2.121)^2$

$\text{Area} \doteq 2.25$

The area of the real garden is about 2.25 m^2.

3. D

A. $\sin B = \frac{b}{c}$

$\neq \frac{a}{c}$

B. $\cos A = \frac{b}{c}$

$\neq \frac{a}{c}$

C. $\tan B = \frac{b}{a}$

$\neq \frac{a}{b}$

D. $\sin A = \frac{a}{c}$

$\cos B = \frac{a}{c}$

4. C

$\cos\theta = \frac{7.1}{7.7}$

$\angle\theta = \cos^{-1}\left(\frac{7.1}{7.7}\right)$

$\angle\theta \doteq 23°$

5. D

$\sin 59.0° = \frac{x}{5.83}$

$x = 5.83 \sin 59°$

$x \doteq 5.00$

The length of AC is about 5.00 cm.

6. A

$MC = BC \div 2$

$MC = 12 \div 2$

$MC = 6$

$\cos 50° = \frac{6}{x}$

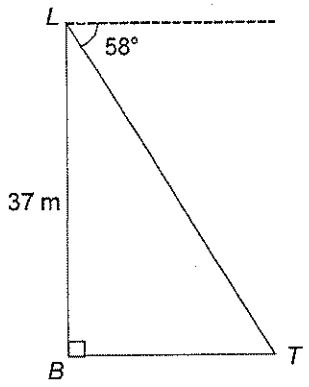
$x = \frac{6}{\cos 50°}$

$x \doteq 9.3$

The length of AC is about 9.3 cm.

7. B

Draw a diagram to model the problem.



$\angle T = 58°$

$\tan 58° = \frac{37}{BT}$

$BT = \frac{37}{\tan 58°}$

$BT \doteq 23$

The truck is about 23 m from the base of the building.

8. C

$\cos F = \frac{2.0}{8.0}$

$\angle F = \cos^{-1}\left(\frac{2.0}{8.0}\right)$

$\angle F \doteq 75.5°$

9. D

Let AB be the base. Determine the height, h.

$\sin 18.5° = \frac{h}{6.33}$

$h = 6.33 \sin 18.5°$

$h \doteq 2.00$

The height is about 2.00 cm.

$\text{Area} = \frac{1}{2}bh$

$\text{Area} \doteq \frac{1}{2}(9.00)(2.00)$

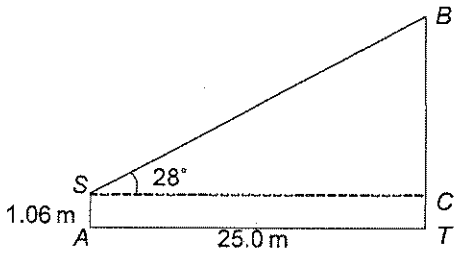
$\text{Area} \doteq 9.00$

The area of the triangle is about 9.0 cm^2.

10. D

Draw a diagram to model the problem.

106 cm = 1.06 m



Determine the length of BC, then determine BT.

$$\tan 28° = \frac{BC}{25.0}$$

$$BC = 25.0 \tan 28°$$

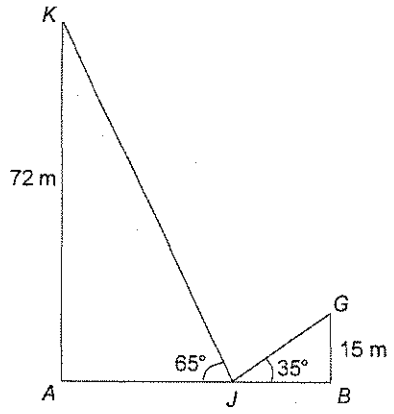
$$BC \doteq 13.3$$

The bird is about 13.3 m above Stephanie's eyes.

The bird is about 13.3 + 1.06 = 14.4 m off the ground.

11. C

Draw a diagram to model the problem.



$$\tan 65° = \frac{72}{AJ}$$

$$AJ = \frac{72}{\tan 65°}$$

$$AJ \doteq 33.6 \text{ m}$$

The distance from Jon to the point below Kristen is about 33.6 m.

$$\tan 35° = \frac{15}{JB}$$

$$JB = \frac{15}{\tan 35°}$$

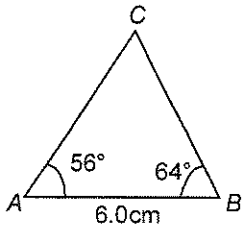
$$JB \doteq 21.4 \text{ m}$$

The distance from Jon to the point below Greg is about 21.4 m.

The width of the gorge is about 33.6 + 21.4 = 55 m.

12. A

Draw a diagram to model the problem.



$$\angle C = 180° - 56° - 64°$$

$$\angle C = 60°$$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{a}{\sin 56°} = \frac{6.0}{\sin 60°}$$

$$a = \frac{6.0 \sin 56°}{\sin 60°}$$

$$a \doteq 5.7$$

The length of a is about 5.7 cm.

13. A

$$\frac{\sin\theta}{81.2} = \frac{\sin 27°}{38.4}$$

$$\sin\theta = \frac{81.2 \sin 27°}{38.4}$$

$$\sin\theta \doteq 0.9600$$

$$\angle\theta = \sin^{-1}(0.9600)$$

$$\angle\theta \doteq 74°$$

14. B

$$a^2 = 5.0^2 + 8.0^2 - 2(5.0)(8.0)\cos 57°$$

$$a^2 = 25 + 64 - 80\cos 57°$$

$$a^2 \doteq 45.43$$

$$a = \sqrt{45.43}$$

$$a \doteq 6.7$$

The length of a is about 6.7 cm.

15. B

$$11.0^2 = 7.0^2 + 15.0^2 - 2(7.0)(15.0)\cos\theta$$

$$121 = 49 + 225 - 210\cos\theta$$

$$210\cos\theta = 49 + 225 - 121$$

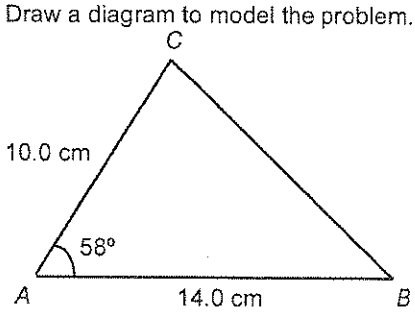
$$\cos\theta = \frac{153}{210}$$

$$\angle\theta = \cos^{-1}\left(\frac{153}{210}\right)$$

$$\angle\theta \doteq 43°$$

16. C

Draw a diagram to model the problem.



$a^2 = 10.0^2 + 14.0^2 - 2(10.0)(14.0)\cos 58°$
$a^2 = 100 + 196 - 280\cos 58°$
$a^2 \doteq 147.623$
$a = \sqrt{147.623}$
$a \doteq 12.2$
The length of a is about 12.2 cm.

$$\frac{\sin B}{10.0} = \frac{\sin 58°}{12.2}$$
$$\sin B = \frac{10.0\sin 58°}{12.2}$$
$\sin B \doteq 0.6951$
$\angle B = \sin^{-1}(0.6951)$
$\angle B \doteq 44°$
$\angle C = 180° - 58° - 44°$
$\angle C = 78°$

17. A

A. Yes because $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ and $a^2 = b^2 + c^2 - 2bc\cos A$.

B. No because $\frac{a}{\sin A} = \frac{b}{\sin B}$, not $\frac{a}{\sin A} = \frac{\sin B}{b}$.

C. No because $a^2 = b^2 + c^2 - 2bc\cos A$, not $a^2 = b^2 + c^2 + 2bc\cos A$.

D. No because $c^2 = a^2 + b^2 - 2ab\cos C$, not $c^2 = a^2 + b^2 + 2ab\cos A$.

18. D

The isosceles triangle can be divided into two congruent right triangles with a hypotenuse of 10 cm and a base that is half the 4-cm side, so 2 cm. Determine the height.

$h^2 = 10^2 - 2^2$
$h^2 = 100 - 4$
$h^2 = 96$
$h = \sqrt{96}$
$h \doteq 9.8$

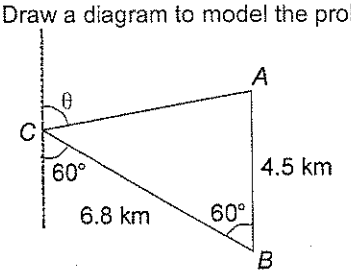
Determine the area of both triangles.

$\text{Area} = 2 \times \frac{1}{2}bh$
$\text{Area} \doteq 2(9.8)$
$\text{Area} \doteq 20$

The area of the isosceles triangle is about 20 cm^2.

19. B

Draw a diagram to model the problem.



$b^2 = 4.5^2 + 6.8^2 - 2(4.5)(6.8)\cos 60°$
$b^2 = 20.25 + 46.24 - 61.2\cos 60°$
$b^2 = 35.89$
$b = \sqrt{35.89}$
$b \doteq 6.0$

The distance back to the starting point is 6.0 km.

$$\frac{\sin C}{4.5} = \frac{\sin 60°}{6.0}$$
$$\sin C = \frac{4.5\sin 60°}{6.0}$$
$\sin C \doteq 0.6495$
$\angle C = \sin^{-1}(0.6495)$
$\angle C \doteq 40.5°$
$\theta = 180° - 60° - 40.5°$
$\theta = 79.5°$

The cyclists needs to ride 6.0 km in the direction N79.5°E.

20. Option A:

The distance down the cliff is 23 m.
The cost for this cable is 12×23 or \$276.

The distance underwater is $\frac{23}{\tan 14°}$ or about 92.248 m.

The cost for this cable is 33×92.248 or about \$3044.18.
The total cost for Option A is about 276 + 3044.18 or \$3320.18.

Option B:

The total length of cable is 39 + 34 + 33 + 61 + 23 or 190 m.
The cost for this cable is 17×190 or \$3230.
The vertical drop from the station to the first tower is 23 – 15 or 8 m.

The angle of depression is $\sin^{-1}\left(\frac{8}{39}\right) \doteq 12°$.

That is two drops of 5° in elevation, so two extra supports are required.
The total cost for Option B is about 3230 + 25 + 25 or \$3280.
Option B is less costly.

21. a) Determine the length of EG knowing that the length of CG is $232.6 \div 2$ or 116.3 m and the hypotenuse of $\triangle EGC$ is 221.2 m.

$EG^2 = EC^2 - GC^2$
$EG^2 = 221.2^2 - 116.3^2$
$EG^2 = 35\ 403.75$
$EG = \sqrt{35\ 403.75}$
$EG \doteq 188.16$

The length of EG is about 188.16 m.

$$\cos EGF = \frac{116.3}{188.16}$$
$$\angle EGF = \cos^{-1}\left(\frac{116.3}{188.16}\right)$$
$\angle EGF \doteq 52°$

b)

$AB^2 = AE^2 + EB^2 - 2(AB)(EB)\cos AEB$
$232.6^2 = 221.2^2 + 221.2^2 - 2(221.2)(221.2)\cos AEB$
$54\ 102.76 = 48\ 929.44 + 48\ 929.44 - 97\ 858.88\cos AEB$
$97\ 858.88\cos AEB = 48\ 929.44 + 48\ 929.44 - 54\ 102.76$
$$\cos AEB = \frac{43\ 756.12}{97\ 858.88}$$
$$\angle AEB = \cos^{-1}\left(\frac{43\ 756.12}{97\ 858.88}\right)$$
$\angle AEB \doteq 63°$